Puzzles of multiplicity

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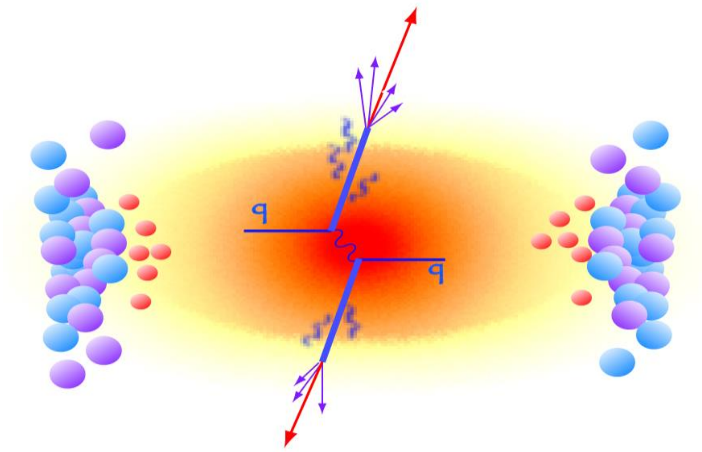
High energy physics began from the study of multiparticle processes. With an increase of the energy of the accelerators, new channels of reactions were opened, in which the number of secondaries increased, and new particles were born. This stimulated the development of phenomenological models and, subsequently, the creation, in particular, of the theory of strong interactions, quantum chromodynamics (QCD). It became possible to increase the speed of data acquisition with the development of electronics, on the basis of which detectors were manufactured.

The main accelerator experiments currently being carried out in high-energy physics are aimed at searching for deviations from the standard model, dark matter particles, and new states of heavy flavors. Close to high-energy physics, relativistic nuclear physics directs its main efforts to the study of quark-gluon matter.



One of the observables in the experiment is the multiplicity - the number of secondary particles. The most predominant among them is the multiplicity of charged particles. Experimenters also restore the multiplicity of neutral particles. The main statistical characteristics of multiplicity are its mean value and a variance. Most often, experimenters analyze events for which the deviation from the average multiplicity does not exceed two average values. Events with much larger multiplicity occur extremely rarely, so it is difficult to collect a large volume of statistics for them, in addition, there are difficulties in processing them.

To understand and describe experiments carried out at accelerators, in particular, distributions in multiplicity, transverse and longitudinal momenta, Monte Carlo generators are built, in which, along with QCD description of quark-gluon interactions, phenomenological models are introduced that describe the hadronization stage (transformation of quarks and gluons into observed hadrons), as well as parton interactions. But when these generators are used to describe the data of hadron experiments in the region of high multiplicity, deviations in the distributions by several orders of magnitude occur.

Despite the fact that the number of Monte Carlo generators is increasing every year, it is still not possible to fully describe the growing amount of experimental data with them. Setting their parameters at the given energy stops working when moving to a higher energy. This evidences a significant misunderstanding of the mechanism of multiple processes. The study of events with the production of a large number of secondary will help to come to a deeper understanding of hadronic interactions, especially the hadronization stage.

In the high multiplicity region, collective phenomena are predicted, including those of a quantum nature. Formations of pion (Bose-Einstein) condensate, Cherenkov radiation of gluons by quarks are those phenomena. In the region of high multiplicity, the longitudinal component of the momentum approaches the transverse component, reaching it. This indicates the disappearance of the leading effect, and the formation of condensate begins in the same region. These and other collective manifestations in the behavior of secondary particles can be studied at the future NICA collider at the SPD facility. These studies will be possible, as it is planned to register events in the absence of any trigger.

Multiparticle production (MP) is one of the most important topics in high energy physics. Using MP we can get more information about the nature of strong interactions and understand deeper the structure of matter. Over the last few years many thorough reviews devoted to MP have been done [1]. Modern accelerators have made it possible to study MP more extensively and in detail. Developing theory of high energy physics quantum chromodynamics (QCD) [2] and a lot of phenomenological models are tested by the process of MP.

Multiparticle processes begin at high energy. Among all producing particles we can observe a lot of hadrons. On one hand we want to know high energy physics, but on the other hand the increase of the inelastic channels makes it difficult to describe this process with customary methods. The situation concerning the history of thermodynamics developing and statistical physics is much the same. Analysis of MP process is carried out using of statistical methods because the number of secondary in e+e− annihilation is large (more than 60) [3]. The consideration of MP begins from the behavior of charged multiplicity.

As it is generally known the multiplicity is the number of secondaries n in process of MP: $\mathbf{A} + \mathbf{B} \rightarrow \mathbf{a_1} + \mathbf{a_2} + \ldots + \mathbf{a_n}$. The multiplicity distribution (MD) $\mathbf{P_n}$ is the ratio of cross-sections $\boldsymbol{\sigma_n}$ to $\boldsymbol{\sigma} = \sum \boldsymbol{\sigma_n}$: $P_n = \boldsymbol{\sigma_n} / \boldsymbol{\sigma}$ **.** This quantity has the following meaning: the probability of producing of n charged particles in this process. We can also construct quantities such as mean values, moments of MD, can study correlations and so on.

Investigation of MP has led to discovery of jets. Jets phenomena can be studied in all processes, where energetic partons are produced. The most common ones are in e+e− annihilation, deep inelastic scattering of e, μ or ν on nucleons and hadron- hadron scattering, involving high-pT particles in final state. Let us consider e+e− -annihilation at high energy. This process is one of the most suitable for the study of MP. In accordance with QCD it can be realized through the production of γ or Z0–boson into two quarks:

$$e^+e^- \rightarrow (\gamma, Z^0) \rightarrow \ q\bar{q} \quad (1)$$

Perturbative QCD can describe the process of fission partons (quarks and gluons) at high energy, because the strong coupling αs is small at that energy. This stage can be called as the stage of cascade. After hard fission, when partons have not high energy, they must be changed into hadrons, which we can observe. On this stage we shouldn’t apply perturbative QCD. Therefore phenomenological models are used for description of hadronization (transformation of quarks and gluons into hadrons) in this case.

The description of the stage quark-gluon cascade by means of perturbative QCD was applied in [4, 5]. Certain features of the predictions at the parton level are expected to be insensitive to details of the hadronization mechanism. They were tested directly by using hadron distributions [6].

The e+e− reaction annihilation is simple for analysis, as the produced state is pure $q\bar{q}$ state. It is usually difficult to determine the quark species on event-by-event basis. The experimental results are averaged over the quark type. Because of confinement the produced quark and gluons fragment into jets of observable hadrons.

The hadronization models are more phenomenological and are built by the experience gained from the study of low p_T hadron collisions. It is usually considered that the producing of hadrons from partons is universal process.

2 Two Stage Model

Parton spectra in QCD quark and gluon jets were studied by Konishi K., Ukawa A. and Veneciano G.[4]. Working at the leading logarithm approximation and avoiding IR divergences by considering finite x, the probabilistic nature of the problem has been established [4].

At the studying of MP at high energy we used idea of A. Giovannini [7] for description of quark-gluon jets as Markov branching processes. Giovannini proposed to interpret the natural QCD evolution parameter **Y**

$$\boldsymbol{Y = \frac{1}{2\pi b} log\left[1 + \alpha_s b log\left(\frac{Q^2}{\mu^2}\right)\right]}, \qquad (2)$$

where $2\pi b = 1/6\,(11N_C - 2N_f)$ for a theory with N_C colours and N_f flavours, as the fickness of the jets and their development as Markov process.

Three elementary processes contribute into QCD jets:

1) gluon fission;
2) quark bremsstrahlung;
3) quark pair creation.

Let AΔY be the probability that gluon in the infinitesimal interval ΔY will convert into twogluons, ˜AΔY be the probability that quark will radiate a gluon, and BΔY be the probability that a quark-antiquark pair will be created from a gluon. A, ˜A, B are assumed to be Y-independent constants and each individual parton acts independently from others, always with the same infinitesimal probability.

Let us define the probability that parton will be transformed into m gluons over a jet of Y in thickness and call it $P_m^P(Y)$. The generating function for a parton jet will be

$$Q^P\,(z, Y) = \sum_{m=0}^{\infty} P_m^P\,(Y) z^m\ . \qquad (3)$$

A.Giovannini constructed system of differential equations and obtained explicit solutions of MD for a parton jet in particular case B = 0 (process of quark pair creation is absent) In the common case $B \neq 0$ MD are similar to particular one [7].

For quark jet explicit solutions are given [7]

$$P_0(Y) = e^{AY},$$

$$P_m(Y) = \frac{\mu(\mu+1)\dots(\mu+m-1)}{m!} e^{-\tilde{A}Y}(1 - e^{-AY})^m, \qquad (4)$$

where $= \frac{\tilde{A}}{A}$. Further the average gluon multiplicity is $\overline{m} = \mu(e^{AY} - 1)$ and the normalized exclusive cross section for producing m gluons from quark is

$$\frac{\sigma_m^q}{\sigma_{tot}} = P_m(Y) =$$

$$= \frac{\mu(\mu-1)\dots(\mu+m-1)}{m!}\left[\frac{\bar{m}}{\bar{m}+\mu}\right]^m\left[\frac{\mu}{\bar{m}+\mu}\right]^\mu. \qquad (5)$$

The generating function (3) will be given by

$$Q^q(z,Y) = \sum_{m=0}^{\infty} z^m P_m(Y) = \left[\frac{e^{-AY}}{1-z(1-e^{-AY})}\right]^\mu. \qquad (6)$$

Eq.(4) is Polya-Egenberger distribution, where μ is non-integer. In Two Stage Model [8] we took (4) for description of cascade stage and added supernarrow binomial distribution for hadronization stage. We chose it based ourselves onanalysis of experimental data in e+e−-annihilation lower 9 GeV. Second correlation moments were negative at this energy. The choice of such distributions was the only could describe experiment.

We suppose that hypothesis of soft colourless is right. We add stage of hadronization to parton stage with aid of its factorization. MD in this process can be written

$$P_n(s) = \sum_m P_m^P P_n^H(m,s)\ , \qquad (7)$$

where P_m^P is MD for partons (4), $P_n^H(m,s)$ – MD or hadrons produced from m partons on the stage of hadronization. Further we will use instead of parameter Y CM(center of masses) energy √s.

In accordance with TSM the stage of hard fission of partons is described by negative binomial distribution (NBD) for quark jet

$$P_m^P(s) = \frac{k_p(k_p+1)\dots(k_p+m-1)}{m!}\left(\frac{\bar{m}}{\bar{m}+k_p}\right)^m\left(\frac{k_p}{\bar{m}+k_p}\right)^{k_p}, \qquad (8)$$

Where $k_p = \frac{\tilde{A}}{A}$, $\tilde{m} = \sum_m mP_m^P$. We neglect of the process (3) quark pair production (B = 0). Two quarks fracture to partons independently of each other. Total MD of two quarks is equal to (7) too. Parameters k_p and m of MD for two joint quark-antiguark jets are doubled, but we use the same designations.

P_m^p and generating function for MD Q^P (s, z) are

$$P_m^P = \frac{1}{m!}\frac{\partial^m}{\partial z^m} Q^P(s,z)|_{z=0}\ , \qquad (9)$$

$$Q_m^P(s,z) = \left[1 + \frac{\bar{m}}{k_p}(1-z)\right]^{-k_p}. \qquad (10)$$

MD of hadrons formed from parton are described in the form [8]

$$P_n^H = C_{k_p}^n\left(\frac{\bar{n}_p^h}{N_p}\right)^n\left(1 - \frac{\bar{n}_p^h}{N_p}(z-1)\right)^{N_p-n}\ , \qquad (11)$$

$C_{k_p}^n$- binomial coefficient) with the generating function

$$Q_p^H = \left[1 + \frac{\bar{n}_p^h}{N_p}(z-1)\right]^{N_p}, \qquad (12)$$

where $\bar{n}_p^h$ and Np (p = q, g) have meaning of average multiplicity and maximum secondaries of hadrons are formed from parton on the stage of hadronization. MD of hadrons in e+e- annihilation are determined by convolution of two stages

$$P_n(s) = \sum_{m=0}^{\infty} P_m^P \frac{\partial^n}{\partial z^n}(Q^H)^{2+m}|_{z=0}, \qquad (13)$$

where 2 + m is total number of partons (two quarks and m gluons).

Further we do the following simplification for the second stage: N_g , considering that probabilities of formation of hadron from quark or gluon are equal. We introduce parameter α = Ng/Nq for distinguishing between hadron jets, created from quark or gluon on the second stage. We also make simplification for designation N = Nq , $\bar{n}^h = \bar{n}_q^h$. Then we get

$$Q_q^H = \left(1 + \frac{\bar{n}^h}{N}(z-1)\right)^N,$$

$$Q_g^H = \left(1 + \frac{\bar{n}^h}{N}(z-1)\right)^{\alpha N}.$$

Introducing in (13) expressions (8), (12) and differentiating on z we obtain MD of hadrons in the process of e+e− annihilation in TSM (two stage model).

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Research program.

Good knowledge of C++ programming language and the ROOT software (http://root.cern.ch) is greeted. Students it is proposed to take part to carry out at the following themes of project:

- Calculations of multiplicity distribution for neutral and charged particles in e+ e- annihilation at high energy in framework of Two Stage Model (TSM).

- Getting acquainted with the work of the main detectors at the HEP experiments: vertex detector, gas trackers, magnetic spectrometer, electromagnetic calorimeter, just to name a few.

- To get the factorial moments F_q and the factorial cumulative moments $K_q$by using MD P_n and/or the generation function G(z), where

$$F_q = \sum_{n=q}^{\infty} n(n-1)\dots(n-q+1)P_n ,$$

$$K_q = F_q - \sum_{i=1}^{q-1} C_{q-i}^{i} K_{q-i} F_i ,$$

Hq = Kq /Fq is their ratio.

- Calculate MD and its average multiplicity for upsilon (bottomonium) decays to three gluons with hadronization.

The number of participating students is 1-4.

The project supervisors: Prof. Elena Kokoulina (Head of the hadron interaction group). Baldin and Veksler Laboratory of High Energy Physics, e-mail: kokoulina@jinr.ru