Introduction to Quantum Computing

TZZ *Report*

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Contents

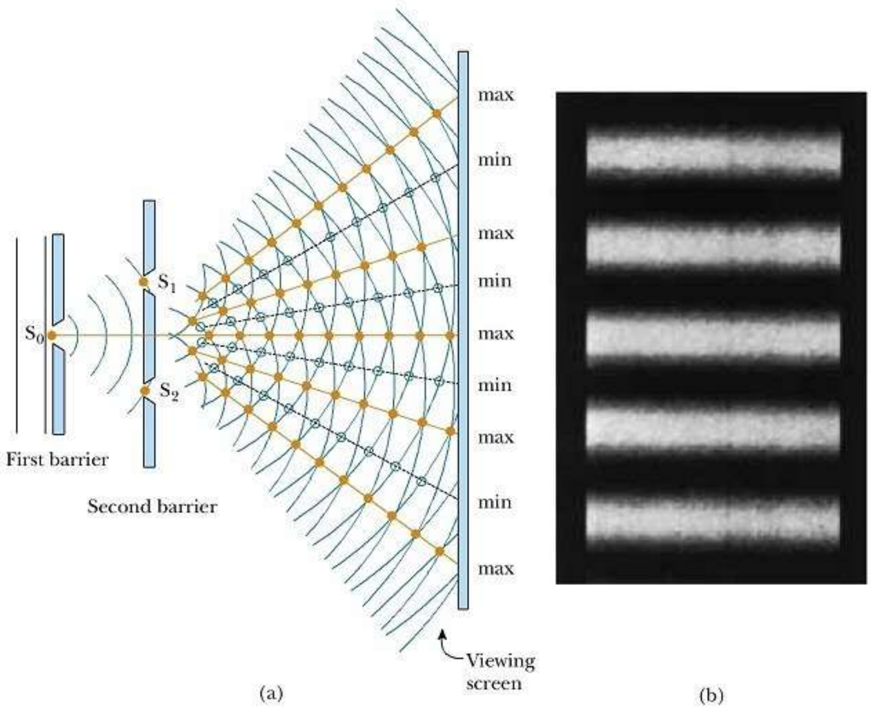
Contents

Abstract

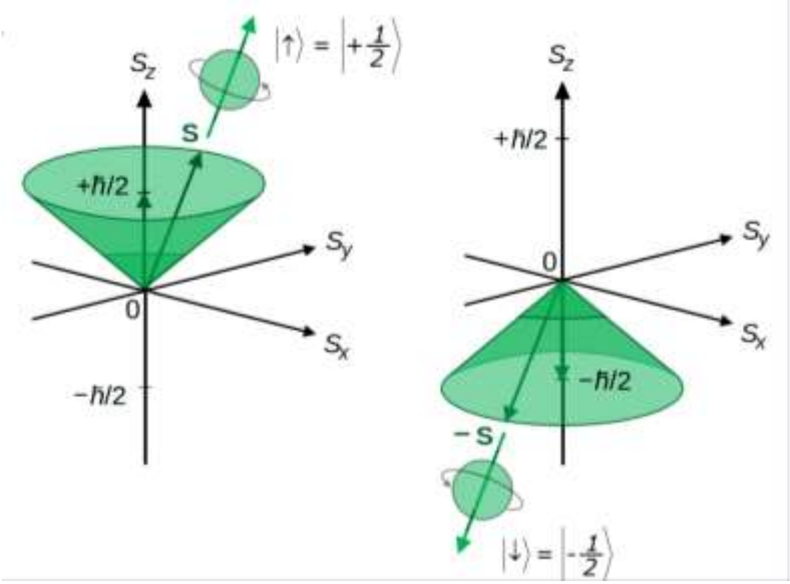
The project presented aimed to introduce the physics behind the world of quantum computing, from the basic concepts of quantum mechanics such as spin mechanics, superpositions of states, the Stern-Gerlach experiment, to the practical applications of qubit measurements, where we looked at the implementation of quantum gates and quantum algorithms such as the Grover algorithm, all this was accomplished with the help of softwares such as CERN's ROOT, and platforms as IBM Q-experience.

Contents

- *From the double slit experiment is known that a quantum state can behave like a wave where these states can interfere constructively and destructively with each other leading to concepts like quantum speedup.*



- *Any two-level quantum system is equivalent to a spin $\frac{1}{2}$ system where the spin is quantized as ↑ or ↓, and from the uncertainty principle we can conclude that in general observables that do not commute can't be measured simultaneously and this is the case for the different components of the spin.*



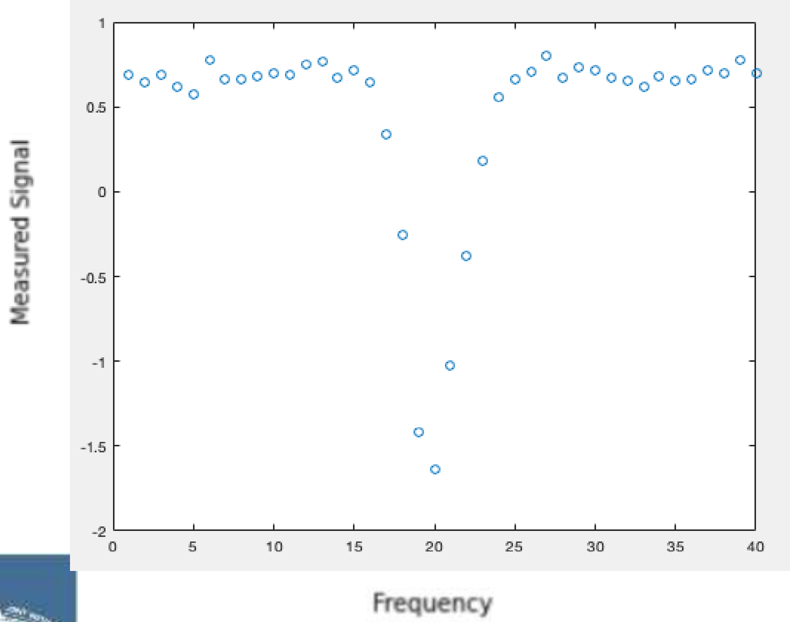
- *Quantum systems allow the superposition of states where the state can be written as a linear superposition of the basis states of the system, where the general state can be given by*

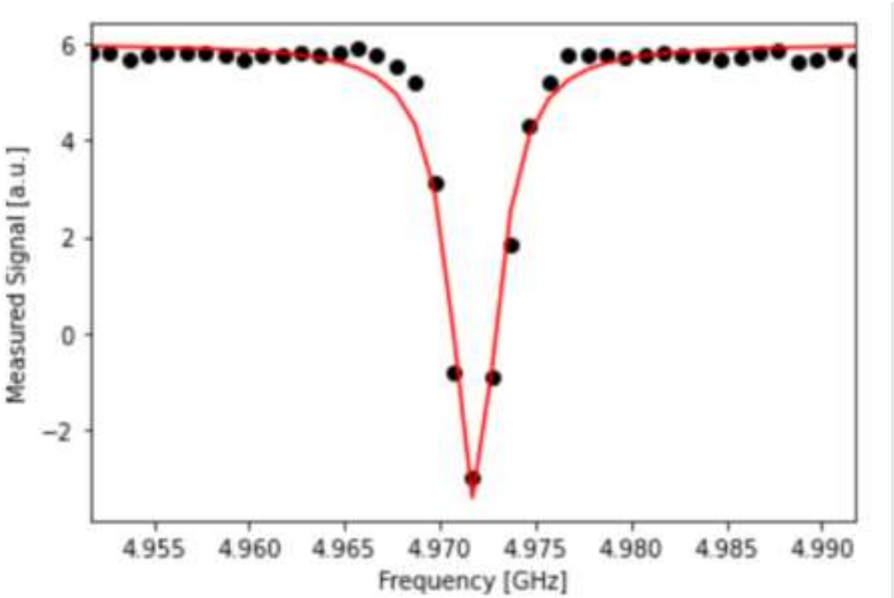
$$|\psi\rangle = c_0\,|0\rangle + c_1\,|1\rangle \qquad c_0, c_1 \in \mathbb{C}$$

Contents

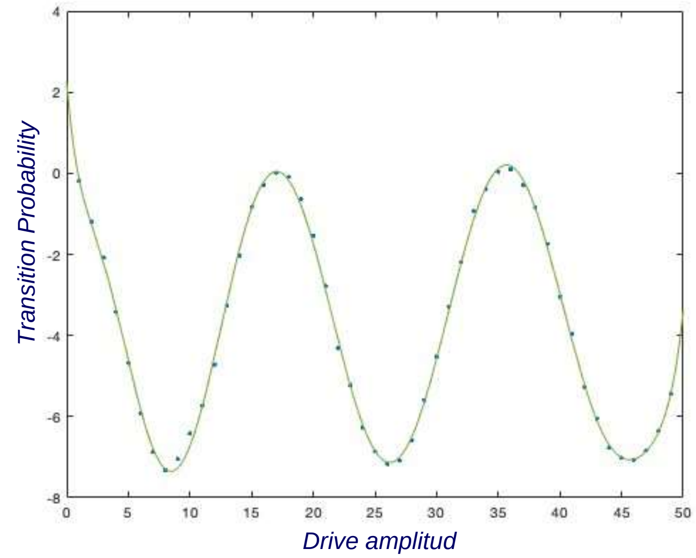
Qubit experiments

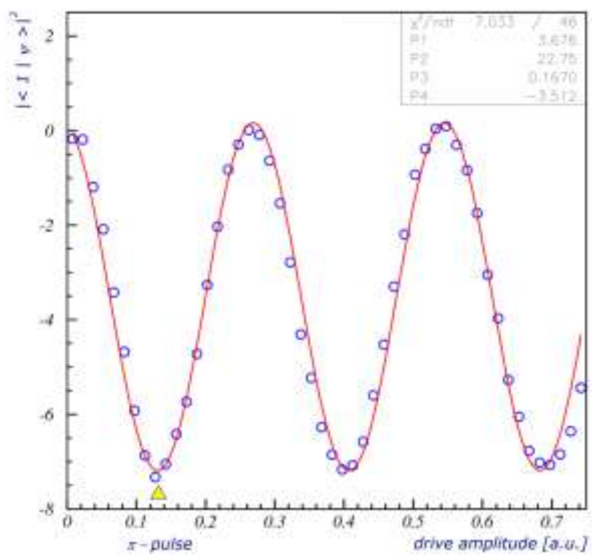
- *Qubit frequency scan: a qubit resonates at a specific frequency f_0 which is the energy difference between the ground and excited state of the qubit $|0\rangle$ and $|1\rangle$. This resonance frequency can be measured by performing equidistant pulse signal measurements over a arbitrary defined range of frequencies.*

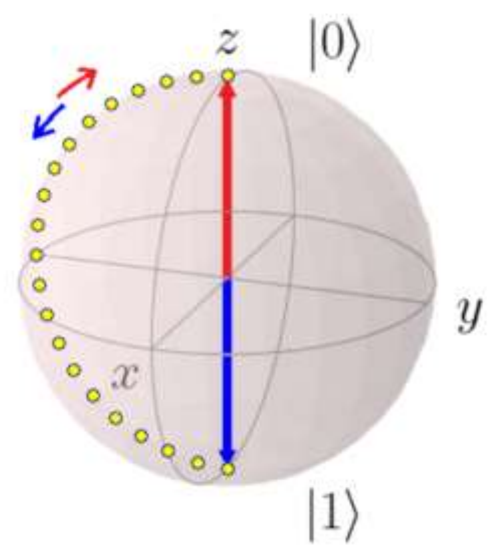




- ***π*-pulse : a *π*-pulse is the pulse that "flips" the state from the state $|0\rangle$ to $|1\rangle$ which visualized on a Bloch sphere would be equivalent to a *π* rotation, and the strength of this pulse can be measured once the resonance frequency mentions in the Qubit frequency scan is found.***

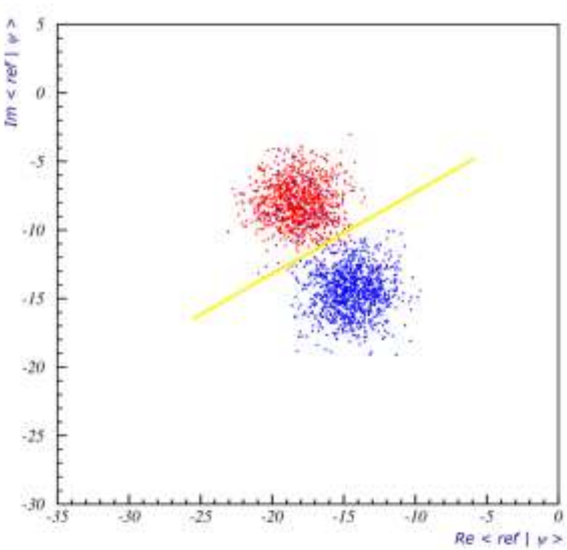


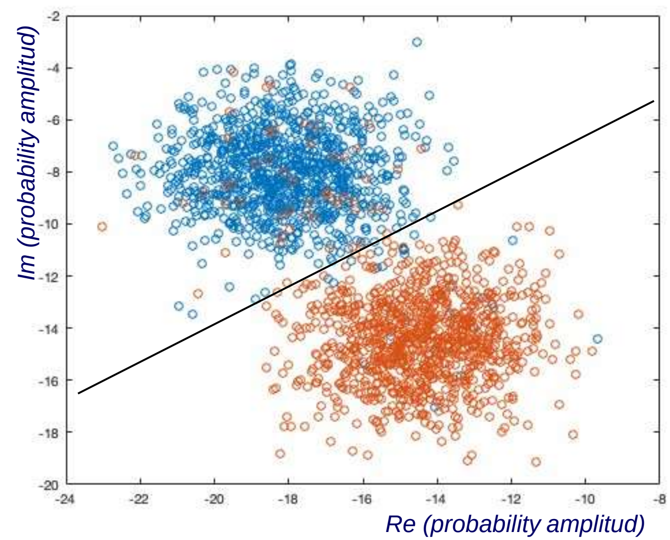




Qubit experiments

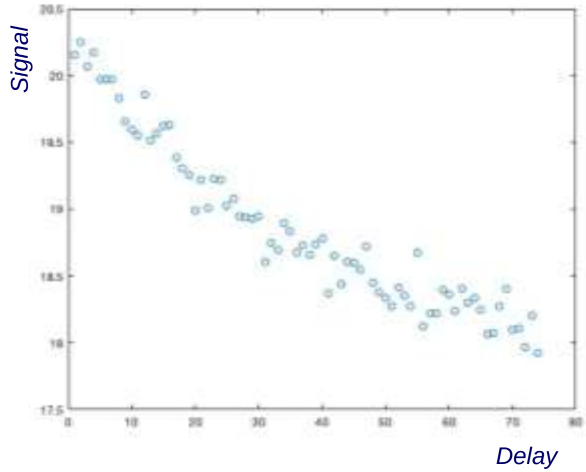
- *0/1 discriminator: once the **π**-pulse has been determined the qubit can be measured with will cause it to collapse into either state, by repeating this measurement and recording and plotting this information we will be able to visualize the populations of each state on a scattered plot and see where the borderline can be found between the clusters.*

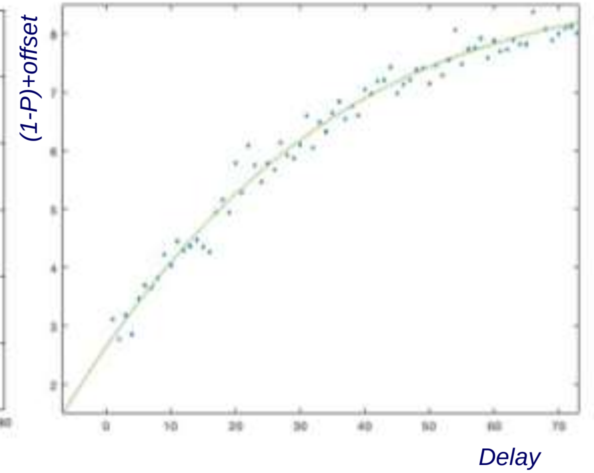


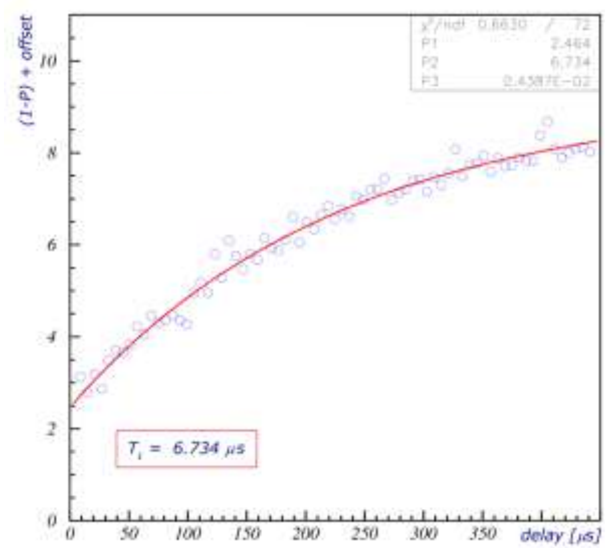


Qubit experiments

- *T1 determination via inversion recovery: T1 is the qubit relaxation time from the excited state to the ground state this happens due to the fact that by nature a qubit has the tendency to be in the lowest energy level and this time can be determined by measuring the signal after the application of a pulse with different delay times which when plotted reflects an exponential decay time.*

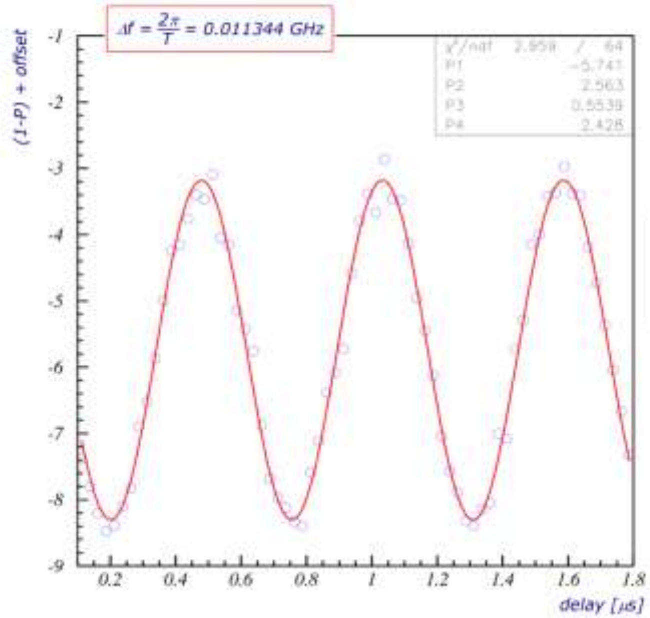


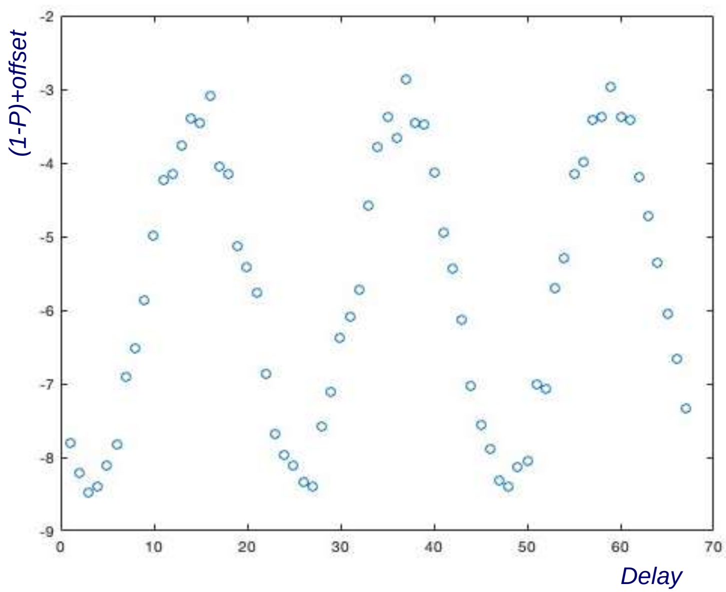


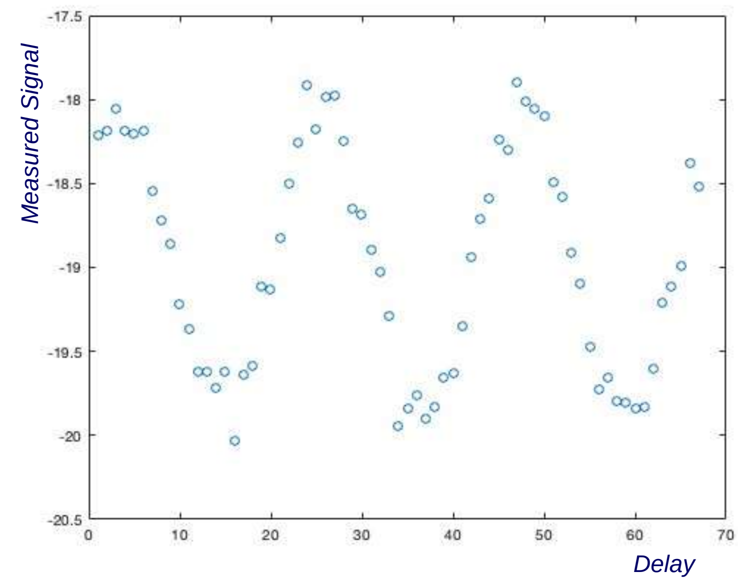


Qubit experiments

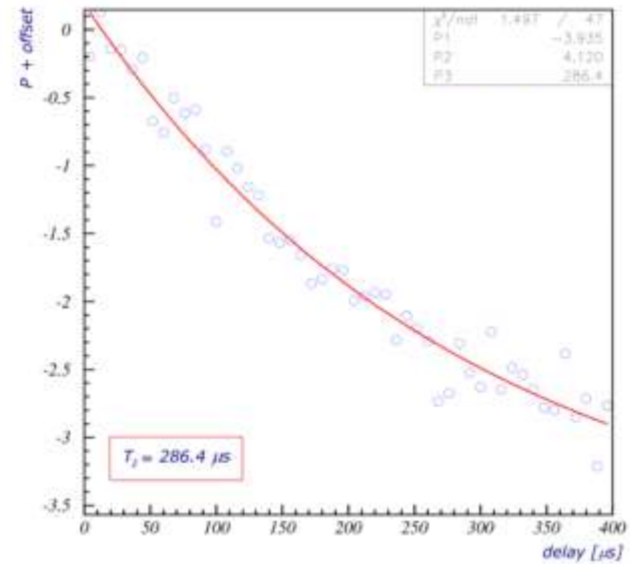
- *Ramsey experiment: during this experiment a **π**/2-pulse is applied and the some we wait some time before applying another **π**/2-pulse then the qubit is measured using frequencies equal to that of the pulses in order to observe the oscillations at the difference in frequencies between this and f_0 .*

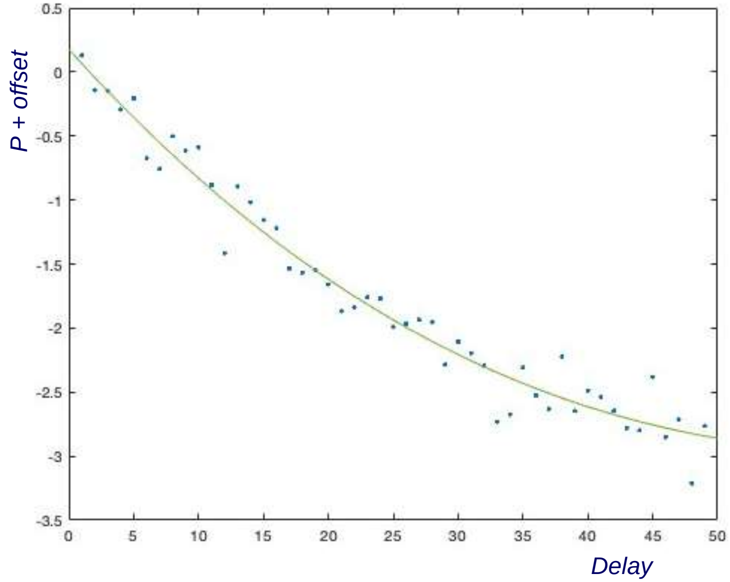


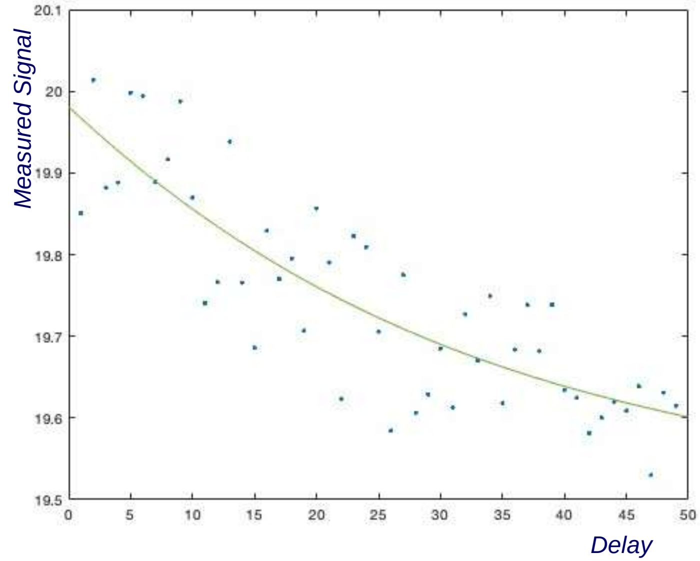




Hahn T2 determination: In order to measure T2 (known as coherence time) we follow the same procedure as with the Ramsey experiment but adding a π *between the two* $\pi/2$ *pulses at time* τ*, this will reverse the phase accumulation generating an echo at a* 2τ *time, the we can measure the decay time of T2.*

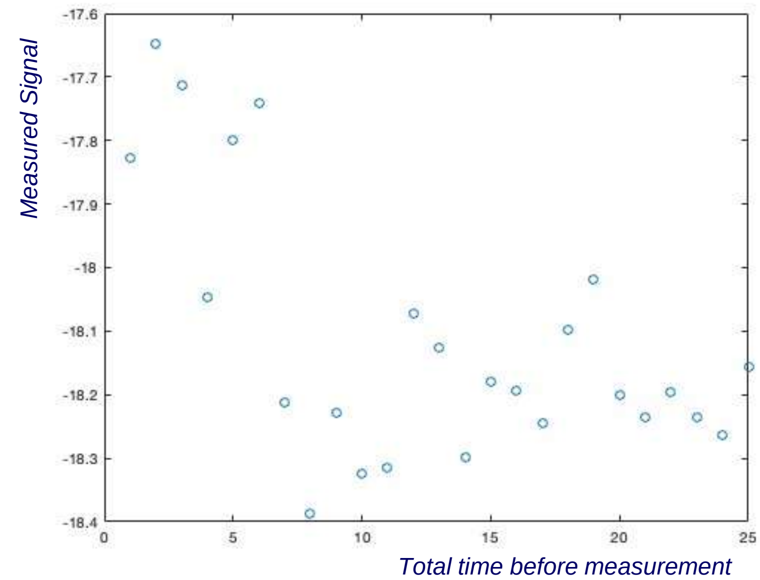


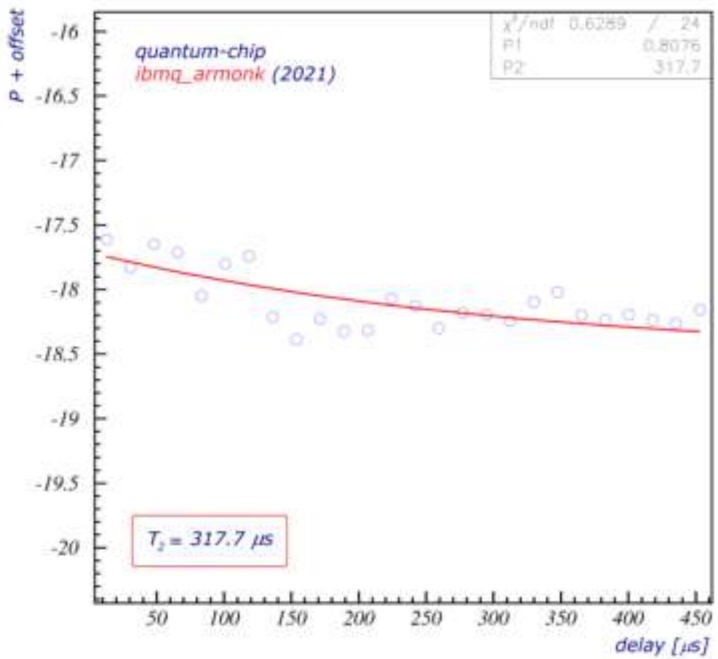


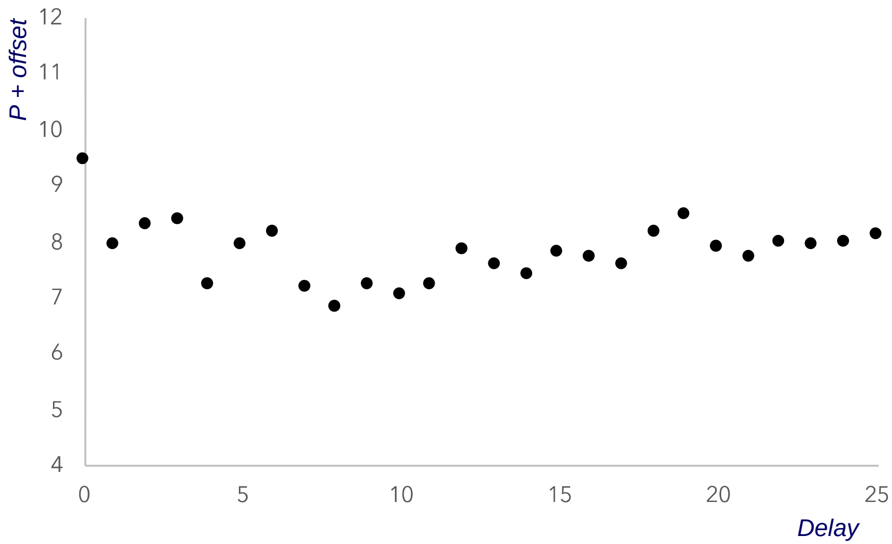


Qubit experiments

- *Dynamic Decoupling: Following the same procedure as with the determination of T2 but applying multiple **π** pulses between the two **π**/2 pulses will allow us to cancel several noise frequencies which will extend the coherence time of from the qubit.*

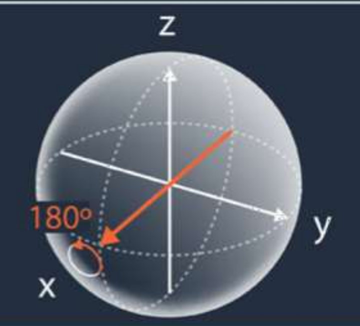
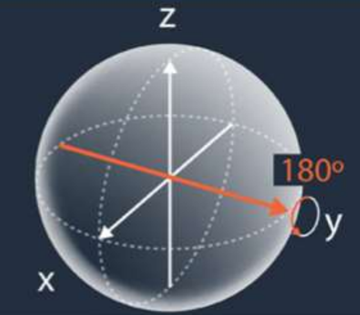
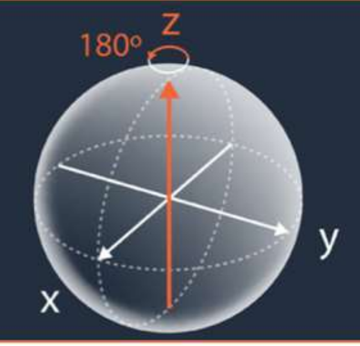


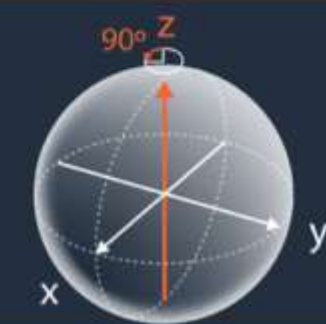




Contents

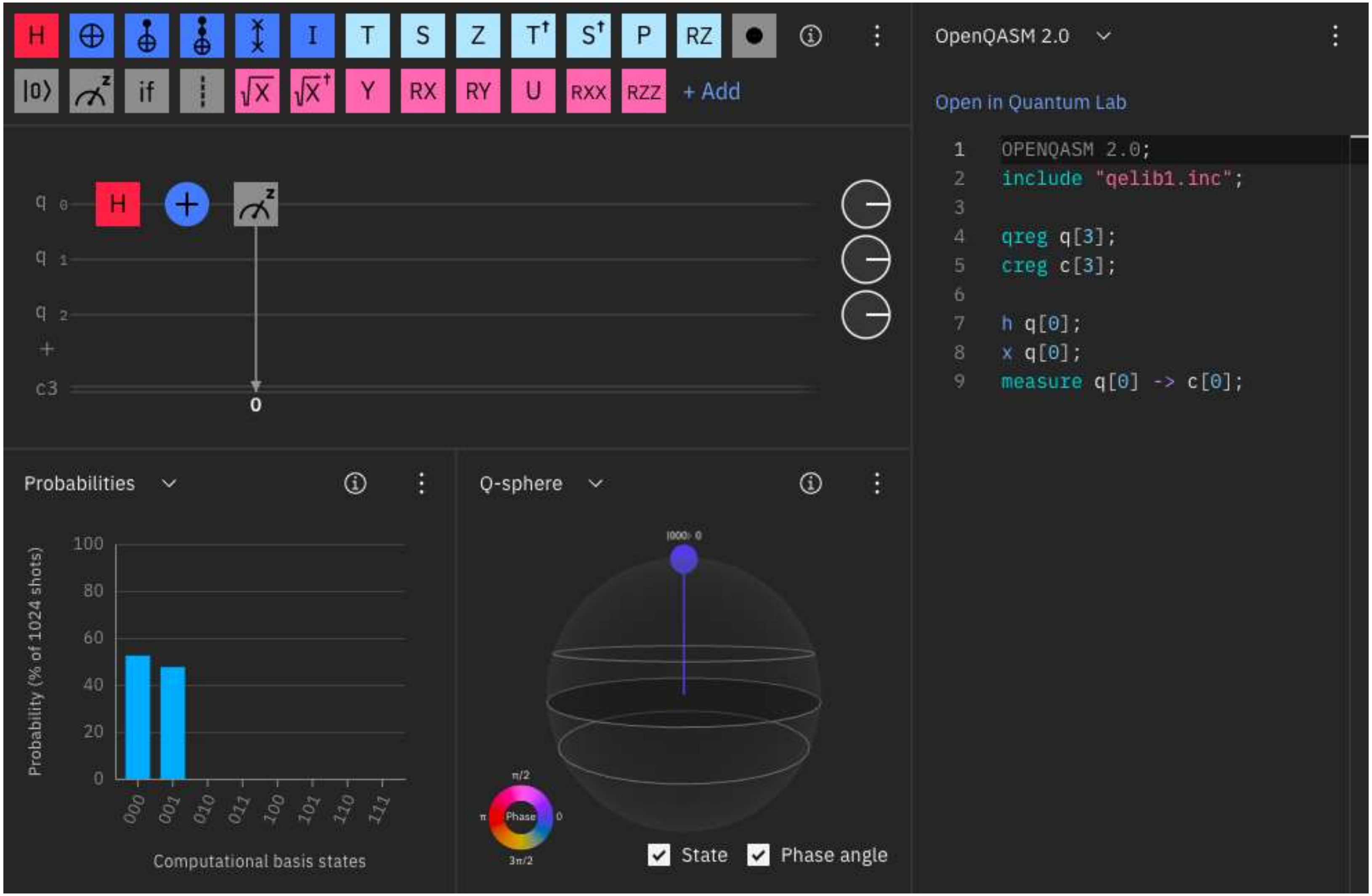
An analogous concept to the classical logic gates are the quantum logic gates which act o the qubit state as operator where the output depends on the input and the gate itself trivially, there exist single cubit gates which act on a single qubit and there exist multiple cubit gates which act on the tensor product of the single qubit states i.e. They act on quantum registers. Some quantum gates act as unitary operators, and when visualized on the Bloch sphere, this unitary operators translate into a rotation that can be characterized by its azimuthal and polar angles.

GATE	CIRCUIT REPRESENTATION	MATRIX REPRESENTATION	TRUTH TABLE	BLOCH SPHERE
X gate: rotates the qubit state by π radians (180º) about the x-axis.	X	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Input / Output: $\|0\rangle$ → $\|1\rangle$; $\|1\rangle$ → $\|0\rangle$	
Y gate: rotates the qubit state by π radians (180º) about the y-axis.	Y	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	Input / Output: $\|0\rangle$ → $i\|1\rangle$; $\|1\rangle$ → $-i\|0\rangle$	
Z gate: rotates the qubit state by π radians (180º) about the z-axis.	Z	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Input / Output: $\|0\rangle$ → $\|0\rangle$; $\|1\rangle$ → $-\|1\rangle$	

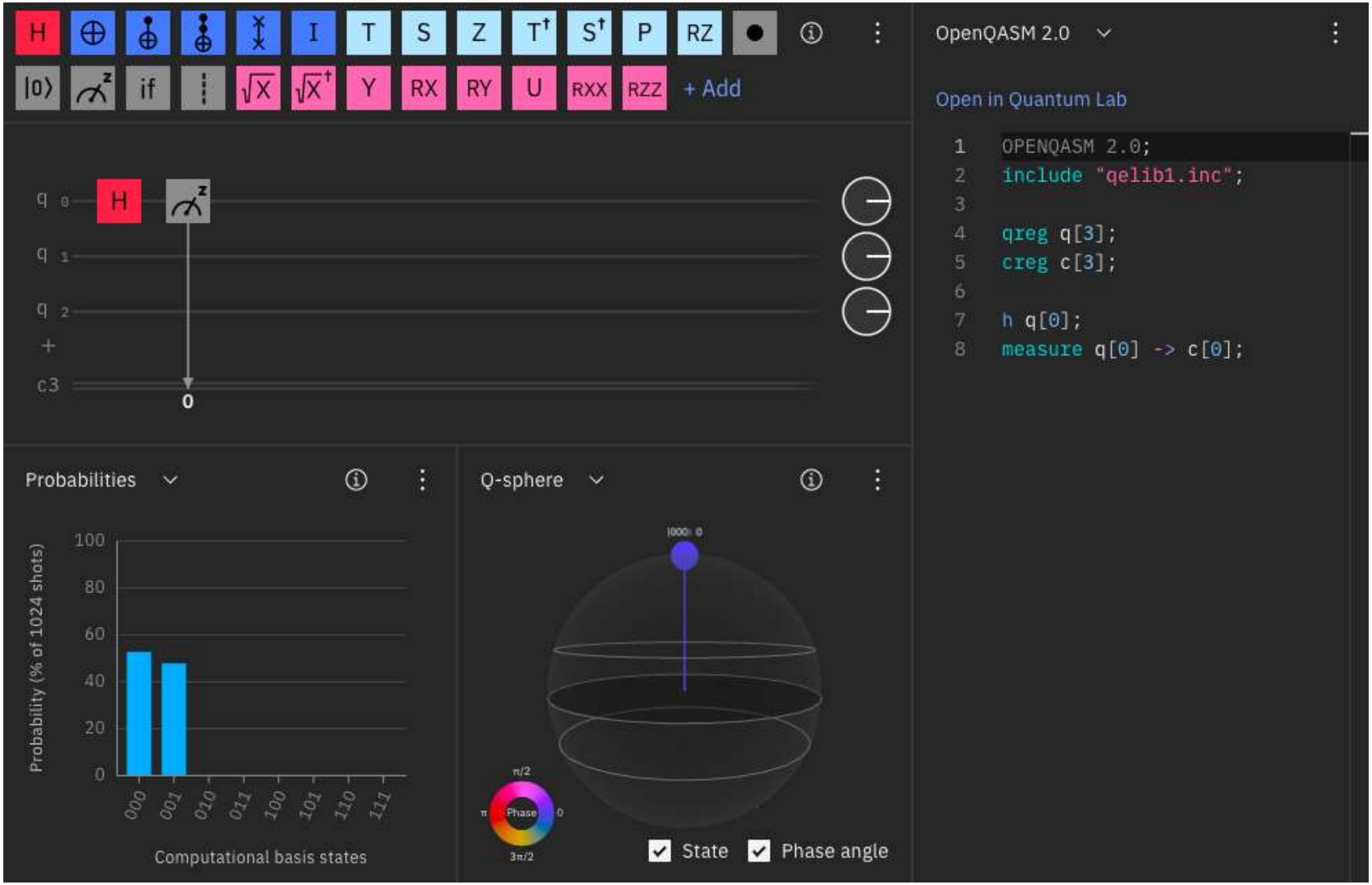
GATE	CIRCUIT REPRESENTATION	MATRIX REPRESENTATION	TRUTH TABLE	BLOCH SPHERE
I Identity-gate: no rotation is performed.	—[I]—	$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Input / Output $\lvert 0\rangle$ → $\lvert 0\rangle$ $\lvert 1\rangle$ → $\lvert 1\rangle$	
S gate: rotates the qubit state by $\frac{\pi}{2}$ radians (90°) about the z-axis.	—[S]—	$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix}$	Input / Output $\lvert 0\rangle$ → $\lvert 0\rangle$ $\lvert 1\rangle$ → $e^{i\frac{\pi}{2}}\lvert 1\rangle$	
T gate: rotates the qubit state by $\frac{\pi}{4}$ radians (45°) about the z-axis.	—[T]—	$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$	Input / Output $\lvert 0\rangle$ → $\lvert 0\rangle$ $\lvert 1\rangle$ → $e^{i\frac{\pi}{4}}\lvert 1\rangle$	

GATE	CIRCUIT REPRESENTATION	MATRIX REPRESENTATION	TRUTH TABLE	BLOCH SPHERE
H gate: rotates the qubit state by π radians (180°) about an axis diagonal in the x-z plane. This is equivalent to an X-gate followed by a $\frac{\pi}{2}$ rotation about the y-axis.	—[H]—	$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	Input / Output $\vert 0\rangle$ → $\frac{\vert 0\rangle + \vert 1\rangle}{\sqrt{2}}$ $\vert 1\rangle$ → $\frac{\vert 0\rangle - \vert 1\rangle}{\sqrt{2}}$	

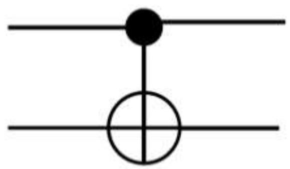
Not gate in the IBM Q-experience lab.

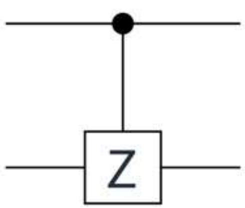


Hadarmard gate in the IBM Q-experience lab.

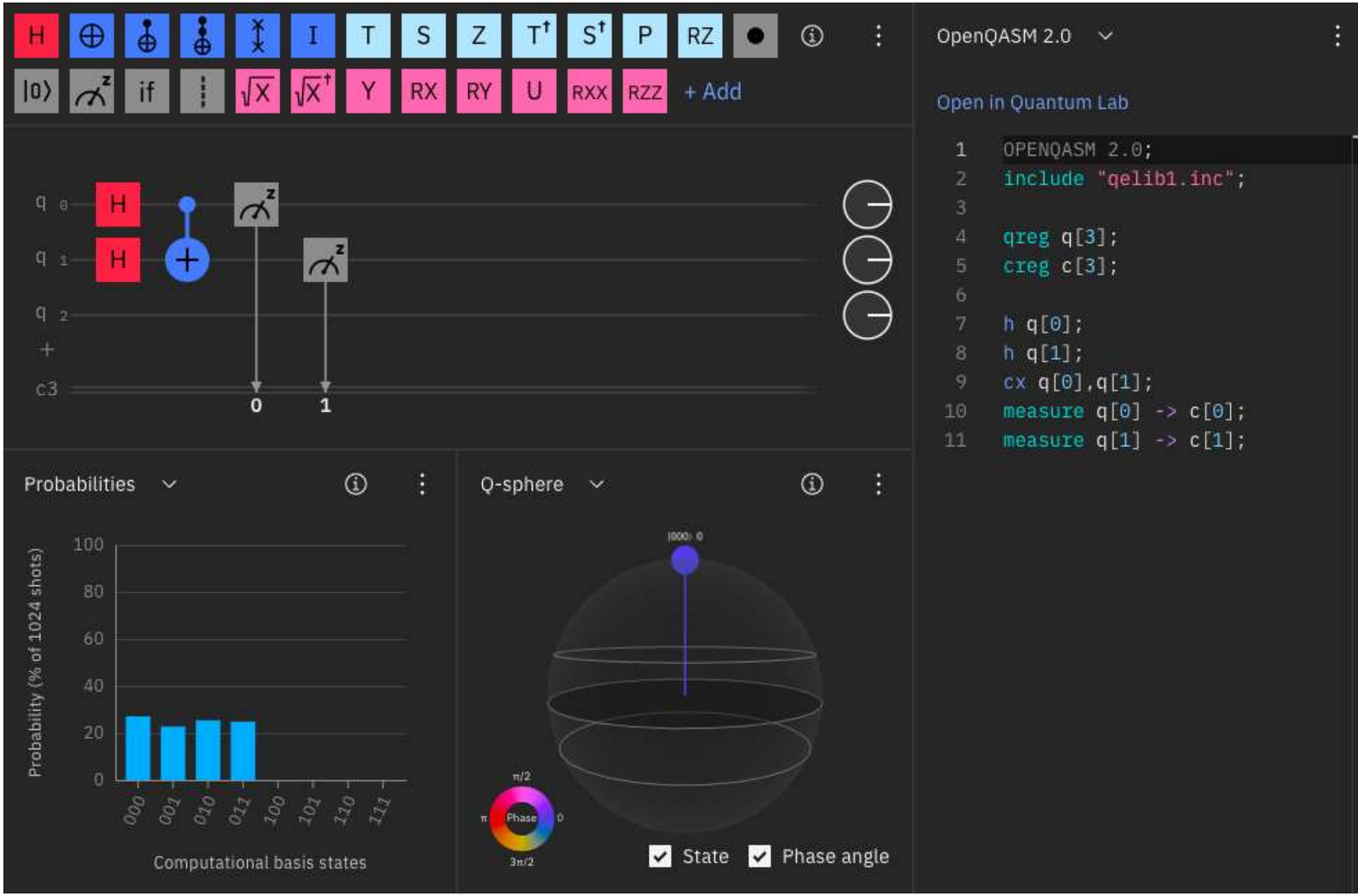


Another type of quantum gate are the controlled gates in which the input are two qubits one works as a controlled qubit which depending on the state itself will determine the action of a single qubit gate on the second qubit called target qubit.

GATE	CIRCUIT REPRESENTATION	MATRIX REPRESENTATION	TRUTH TABLE
Controlled-NOT gate: apply an X-gate to the target qubit if the control qubit is in state $\|1\rangle$		$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	Input → Output $\|00\rangle$ → $\|00\rangle$ $\|01\rangle$ → $\|01\rangle$ $\|10\rangle$ → $\|11\rangle$ $\|11\rangle$ → $\|10\rangle$

GATE	CIRCUIT REPRESENTATION	MATRIX REPRESENTATION	TRUTH TABLE
Controlled-phase gate: apply a Z-gate to the target qubit if the control qubit is in state $\|1\rangle$		$\text{CPHASE} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	Input → Output $\|00\rangle$ → $\|00\rangle$ $\|01\rangle$ → $\|01\rangle$ $\|10\rangle$ → $\|10\rangle$ $\|11\rangle$ → $-\|11\rangle$

Controlled not gate when applied to superposition of qubits

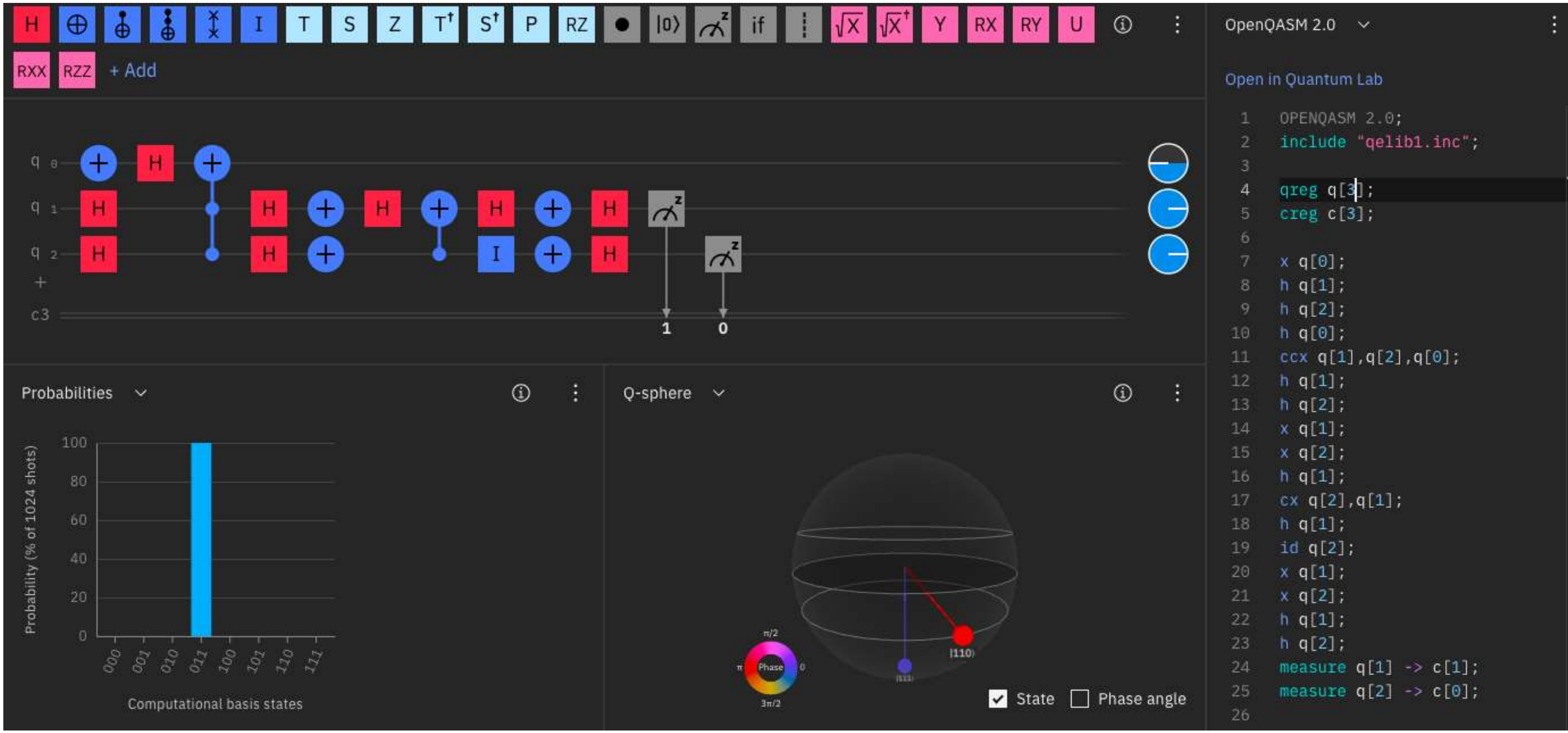


Contents

Quantum algorithm

Quantum algorithms are a computational tool which takes advantage of the features of quantum states to solve problems significantly faster than a classical algorithm would, an example of these type of algorithms is the Groover's algorithm which basically searches from a list of N terms one specific one, while a classical algorithm would require N/2 iterations to do so, this quantum algorithm reduced this quantity to √N.

Grover's algorithm.



Conclusions

Personal opinions

- During this wave we studied basic concepts of quantum computing as well as the implementation of this concepts, personally, the field of quantum computation has always been of my interest so working on this project represented for me an opportunity to acquire more knowledge and experience in this field that seems so interesting and innovative to me, as well as to reinforce the knowledge I had have gained during my education years, other than that it also introduced the application of platforms such as IBM Q-experience which granted access for us to do quantum computations when carrying out the different experiments. Finally, I would like to express my gratitude to the Joint Institute of Nuclear Research for the opportunity as well as to Dr. Mihai for his time and effort during these weeks.