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Puzzles of Multiplicity

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1 Abstract

Multiparticle production (MP) stands as a crucial subject within high-energy physics, offering valuable insights into the fundamental aspects of strong interactions and providing a deeper understanding of the structure of matter. Multiparticle processes occur at high energy levels, with a notable presence of hadrons among the produced particles. The investigation of MP has led to the discovery of jets, phenomena that can be studied in processes producing energetic partons. One prevalent example of such processes is electron-positron annihilation, which we consider for the analysis of MP at high energy. The challenge arises at higher energy levels due to the increase of inelastic channels, making it difficult to describe MP using conventional methods. To tackle this, statistical methods are employed in the analysis of MP processes, especially in the context of e^+e^- annihilation where the number of secondary particles is large. A useful approach involves treating QCD jets as Markov branching processes, enabling a probabilistic description of the evolution of parton showers within hadrons. This stochastic approach provides clear and comprehensive solutions for the parton multiplicity distribution.

2 Introduction

Multiparticle production (MP) constitutes a significant domain within high-energy physics, with modern accelerators enabling thorough investigations into these processes. The study of multiparticle production serves as a vital testing ground for Quantum Chromodynamics (QCD), along with various phenomenological models.

Current accelerator experiments conducted within high-energy physics are primarily focused on uncovering potential deviations from the standard model, identifying dark matter particles, and discovering new states of heavy flavors. Concurrently, in the realm of relativistic nuclear physics, closely related to high-energy physics, significant efforts are directed towards exploring quark-gluon matter.

In such experiments, a key observable is the multiplicity, a measure denoting the number of secondary particles produced in the process of multiparticle production represented as

$$a + b \rightarrow c_1 + c_2 + c_3 + ... + c_n$$

where the focus lies not only on the resultant particles but also on the statistical attributes characterizing their multiplicity phenomenon.

To grasp the intricacies of multiplicity, we rely on statistical metrics such as the mean value and variance. These metrics play a pivotal role in understanding the distribution of secondary particles. For instance, the probability mass function

$$P_k = \frac{N_k}{\Sigma_{i=k} N_i} \tag{1}$$

sheds light on the likelihood of observing k secondary particles. The average multiplicity

$$\langle n \rangle = \Sigma k P_k = \frac{\Sigma k N_k}{\Sigma N_k} \tag{2}$$

serves as a valuable metric, calculated as the weighted sum of multiplicities, taking into account their respective probabilities. Furthermore, to capture the spread or dispersion of the multiplicity distribution, we introduce the variance, expressed as the difference between the average squared multiplicity and the square of the average multiplicity

$$D^2 = \langle n^2 \rangle - \langle n \rangle^2 \tag{3}$$

The annihilation process of electron-positron pairs, Figure 1, stands out as one of the most effective means to investigate MP phenomenon. When an electron collides with a positron, they can annihilate into either a virtual photon or a Z^0 boson. Both the virtual photon and the Z^0 boson subsequently decay into pairs of fermions and antifermions, specifically quarks and antiquarks.

$$e^+e^- \to (\gamma/Z^0) \to q\bar{q}$$

This leads to the emission of gluons and the splitting of gluons into quark-antiquark pairs, resulting in a parton shower. This is commonly referred to as the cascade stage. Perturbative QCD is effective in explaining this stage of parton fission at high energy levels, as the strong coupling constant α_s is small at this energy. The subsequent stage, where partons no longer have high energy, involves hadronization which encapsulates the transformation of quarks and gluons into observable hadrons. In this stage, perturbative QCD is not applicable, prompting the use of phenomenological models to describe hadronization. The process concludes with the formation of hadrons and their potential decays.

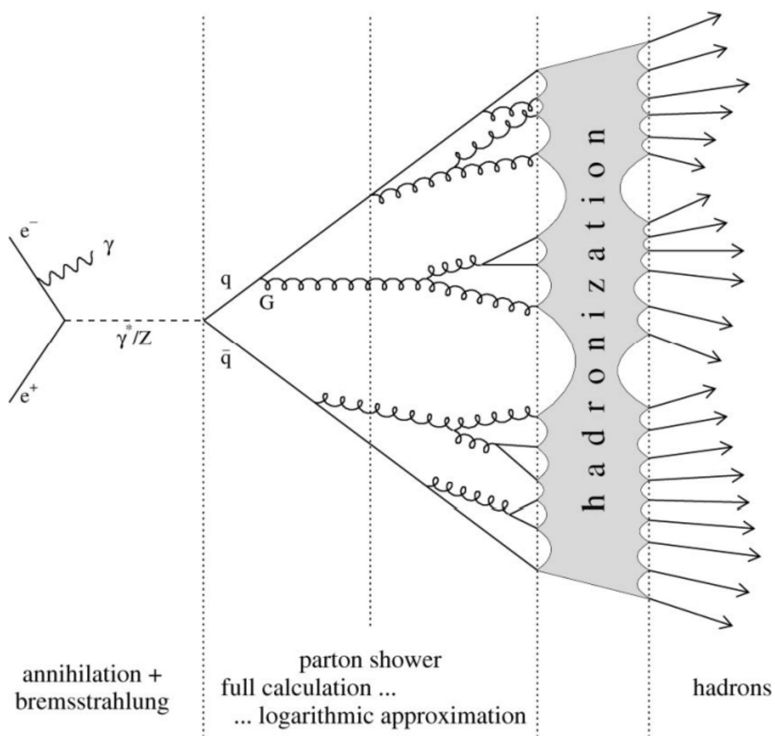

Figure 1: Diagram of e^+e^- annihilation

This comprehensive journey, from the annihilation of electron-positron pairs to the emergence of observable hadrons, provides valuable insights into multiparticle production, highlighting the need for an alternative method to describe the phenomenon.

3 QCD jets as Markov branching processes

3.1 Elementary Processes in QCD Jets

To study multiparticle production at high energy, we consider QCD jets as Markov branching processes. We interpret the natural QCD evolution parameter

$$Y = \frac{1}{2\pi b} ln \left[1 + \alpha b \; ln \left(\frac{Q^2}{\mu^2} \right) \right], \tag{4}$$

where $2\pi b = \frac{1}{6}(11N_c - 2N_f)$ for a theory with N_C colors and N_f flavors, as the thickness value of a quark or a gluon that gives rise to a gluon or a quark jet.

There are three main elementary processes that contribute to the overall gluon or quark distribution inside QCD jets with different weights:

- gluon fission: $g \to g + g$, with $A\Delta Y$ denoting the probability that a gluon in the infinitesimal interval ΔY will transform into two gluons
- quark bremsstrahlung: $q \to q + g$, with $\tilde{A}\Delta Y$ denoting the probability that a quark in the infinitesimal interval ΔY will radiate a gluon with the quark continuing on its original trajectory with modified energy and momentum
- quark pair creation: $g \to q + \bar{q}$, with $B\Delta Y$ denoting the probability that a quark-antiquark pair in the infinitesimal interval ΔY will be created from a gluon

We assume that A, $\tilde{A}$, and B are Y-independent constants and that each individual parton acts independently from the others, always with the same infinitesimal probability.

3.2 Generating Functions for Jet Evolution

We introduce the infinitesimal generating functions for gluon jet and quark jet, respectively, as

$$w^{(g)}(u_g, u_q) = \sum_{m_g, m_q = 0}^{\infty} a^{(g)}_{m_g, m_q} u_g^{m_g} u_q^{m_q} = (-A - B)u_g + Au_g^2 + Bu_q^2 \tag{5}$$

$$w^{(q)}(u_g, u_q) = \sum_{m_g, m_q = 0}^{\infty} a^{(q)}_{m_g, m_q} u_g^{m_g} u_q^{m_q} = -\tilde{A}u_q + \tilde{A}u_q u_g \tag{6}$$

Let us define $P_{m_g,m_q;n_g,n_q}(Y)$ to be the probability that m_g gluons and m_q quarks will be transformed into n_g gluons and n_q quarks respectively over a jet of thickness Y. It then follows that the probability generating function for a gluon jet is

$$G(u_g, u_q; Y) = \sum_{n_g,n_q=0}^{\infty} P_{1,0;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} \tag{7}$$

and the probability generating function for a quark jet is

$$Q(u_g, u_q; Y) = \sum_{n_g,n_q=0}^{\infty} P_{0,1;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} \tag{8}$$

When considering the evolution of the total parton population (consisting of both gluons and quarks) through thickness Y, a probabilistic perspective allows us to treat this population as if it were made up of independent sub-populations. Each of these sub-populations behaves as if it originated from a single initial quark or gluon. In essence, the overall evolution can be viewed as a sum of independent parton populations, each starting with one quark or gluon. This can be expressed mathematically as follows:

$$\sum_{n_g,n_q}^{\infty} P_{m_g,m_q;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} = \left[G(u_g, u_q; Y)\right]^{m_g} \left[Q(u_g, u_q; Y)\right]^{m_q} \tag{9}$$

Since the process is homogeneous in Y, the transition probabilities obey Chapman- Kolmogorov equations:

$$P_{m_g,m_q;n_g,n_q}(Y + Y') = \sum_{l_g,l_q}^{\infty} P_{m_g,m_q;l_g,l_q}(Y) P_{l_g,l_q;n_g,n_q}(Y') \tag{10}$$

For a gluon jet, we get

$$P_{0,1;n_g,n_q}(Y + Y') = \sum_{l_g,l_q}^{\infty} P_{1,0;l_g,l_q}(Y) P_{l_g,l_q;n_g,n_q}(Y') \tag{11}$$

and for a quark jet, we get

$$P_{1,0;n_g,n_q}(Y + Y') = \sum_{l_g,l_q}^{\infty} P_{0,1;l_g,l_q}(Y) P_{l_g,l_q;n_g,n_q}(Y') \tag{12}$$

From (9) and (10), we get

$$G(u_g, u_q; Y + Y') = G[G(u_g, u_q; Y'), Q(u_g, u_q; Y'); Y] \tag{13}$$

$$Q(u_g, u_q; Y + Y') = Q[G(u_g, u_q; Y'), Q(u_g, u_q; Y'); Y] \tag{14}$$

From (5)-(8), we can see that

$$G(u_g, u_q; \Delta Y) = u_g + w^{(g)}(u_g, u_q)\Delta Y + O(\Delta Y) \tag{15}$$

$$Q(u_g, u_q; \Delta Y) = u_q + w^{(q)}(u_g, u_q)\Delta Y + O(\Delta Y) \tag{16}$$

Substituting (15) and (16) into (13) and (14), while substituting Y' with ΔY, then dividing both sides by ΔY and letting $\Delta Y \to 0$, we obtain

$$\frac{\partial G(u_g, u_q; Y)}{\partial Y} = \frac{\partial G}{\partial u_g} w^{(g)}(u_g, u_q) + \frac{\partial G}{\partial u_q} w^{(q)}(u_g, u_q) \tag{17}$$

$$\frac{\partial Q(u_g, u_q; Y)}{\partial Y} = \frac{\partial Q}{\partial u_g} w^{(g)}(u_g, u_q) + \frac{\partial Q}{\partial u_q} w^{(q)}(u_g, u_q) \tag{18}$$

3.3 Differential Equations for QCD Jets

We can recognize the forward Kolmogorov equations for the generating functions of the transition probability $P_{m_g,m_q;n_g,n_q}(Y)$ in (17) and (18) The corresponding backward Kolmogorov equations follow from (13) and (14)

$$\frac{\partial G}{\partial Y} = w^{(g)}[G(u_g, u_q; Y), Q(u_g, u_q; Y)] \tag{19}$$

$$\frac{\partial Q}{\partial Y} = w^{(q)}[G(u_g, u_q; Y), Q(u_g, u_q; Y)] \tag{20}$$

Substituting (5) and (6) into (19) and (20), we obtain

$$\frac{\partial G}{\partial Y} = -AG + AG^2 - BG + BQ^2 \tag{21}$$

$$\frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG \tag{22}$$

We can find the probability for a gluon or a quark to produce n_g gluons and n_q quarks in the interval $Y + \Delta Y$ through the main elementary processes. For a gluon jet

$$\begin{aligned} P_{1,0;n_g,n_q}(Y + \Delta Y) =& (1 - \tilde{A}n_q\Delta Y - An_g\Delta Y - Bn_g\Delta Y)P_{1,0;n_g,n_q}(Y) \\ &+\tilde{A}n_q\Delta Y P_{1,0;n_g-1,n_q}(Y) + A(n_g - 1)\Delta Y P_{1,0;n_g-1,n_q}(Y) \\ &+B(n_g + 1)\Delta Y P_{1,0;n_g+1,n_q-2}(Y) + O(\Delta Y) \end{aligned} \tag{23}$$

Dividing both sides by ΔY and letting $\Delta Y \to 0$, we obtain the following system of differential equations

$$\begin{aligned}\frac{dP_{1,0;n_g,n_q}(Y)}{dY} =&(-\tilde{A}n_q - An_g - Bn_g)P_{1,0;n_g,n_q}(Y) \\ &+\tilde{A}n_q P_{1,0;n_g-1,n_q}(Y) + A(n_g-1)P_{1,0;n_g-1,n_q}(Y) \\ &+B(n_g+1)P_{1,0;n_g+1,n_q-2}(Y)\end{aligned} \tag{24}$$

For the gluon exclusive cross-sections in a gluon jet or a quark jet, we respectively have the following

$$\frac{dP_{1,0;n_g,0}(Y)}{dY} =(-A-B)n_g P_{1,0;n_g,0}(Y) + A(n_g-1)P_{1,0;n_g-1,0}(Y) \tag{25}$$

$$\begin{aligned}\frac{dP_{0,1;n_g,1}(Y)}{dY} =& -\tilde{A}P_{0,1;n_g,1}(Y) - (B+A)n_g P_{0,1;n_g,1}(Y) \\ &+\tilde{A}P_{0,1;n_g-1,1}(Y) + A(n_g-1)P_{0,1;n_g-1,1}(Y)\end{aligned} \tag{26}$$

The corresponding generating functions are

$$\frac{\partial G}{\partial Y} = -AG + AG^2 - BG \tag{27}$$

$$\frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG \tag{28}$$

3.4 Explicit solutions in particular cases

While obtaining explicit solutions in terms of the generating functions, (27) and (28), or of the exclusive cross sections, (23), is generally challenging, it is possible to derive approximate solutions for specific cases. These approximations prove to be particularly intriguing and contribute to a deeper comprehension of the overall problem.

We make the approximation $B = 0$, $A \neq \tilde{A} \neq 0$, meaning that we don't allow gluons to split into quark-antiquark pairs. In other words, from the definition of B, there is no room for flavors in the theory. Then (27) and (28) become

$$\frac{\partial G}{\partial Y} = -AG + AG^2 \tag{29}$$

$$\frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG \tag{30}$$

The gluon exclusive cross-sections in a gluon jet or a quark jet satisfy the following

$$\frac{dP_{1,0;n_g,0}(Y)}{dY} = -An_g P_{1,0;n_g,0}(Y) + A(n_g-1)P_{1,0;n_g-1,0}(Y) \tag{31}$$

$$\begin{aligned}\frac{dP_{0,1;n_g,1}(Y)}{dY} =& -\tilde{A}P_{0,1;n_g,1}(Y) - An_g P_{0,1;n_g,1}(Y) \\ &+\tilde{A}P_{0,1;n_g-1,1}(Y) + A(n_g-1)P_{0,1;n_g-1,1}(Y)\end{aligned} \tag{32}$$

with the following initial conditions

$$P_{1,0;1,0}(0) = 1,\ P_{1,0;n_g,0}(0) = 0,\ \forall n_g > 1 \tag{33}$$

$$P_{0,1;0,1}(0) = 1,\ P_{0,1;n_g,1}(0) = 0,\ \forall n_g \geq 1 \tag{34}$$

3.4.1 Gluon Jet

From (31) and (33), we obtain

$$P_{1,0;1,0}(Y) = e^{-AY} \tag{35}$$

$$P_{1,0;n_g,0}(Y) = e^{-AY}(1-e^{-AY})^{n_g-1}, \tag{36}$$

where the average gluon multiplicity is $\langle n_g \rangle = e^{AY}$.

The normalized exclusive cross-section for producing n_g gluons is

$$\frac{\sigma^{(g,0)}_{n_g}}{\sigma_{tot}} = P_{1,0;n_g,0}(Y) = \frac{1}{\langle n_g \rangle}\left(1 - \frac{1}{\langle n_g \rangle}\right)^{n_g-1} \tag{37}$$

which corresponds to a Furry-Yule distribution. The variance is

$$D^2 = e^{AY}(e^{AY}-1) \tag{38}$$

Thus we can obtain the second correlative moment to be

$$f_2 = e^{2AY} - 2e^{AY} \tag{39}$$

and the corresponding generating function is

$$G = \sum_{n_g=0}^{\infty} u_g^{n_g} P_{1,0;n_g,0}(Y) = \frac{u_g e^{-AY}}{1-u_g(1-e^{-AY})} \tag{40}$$

3.4.2 Quark Jet

From (32) and (34), we obtain

$$P_{0,1;0,1}(Y) = e^{-\tilde{A}Y} \tag{41}$$

$$P_{0,1;n_g,1}(Y) = \frac{\mu(\mu+1)...(\mu+n_g-1)}{n_g!} e^{-\tilde{A}Y}(1-e^{-AY})^{n_g}, \tag{42}$$

where $\mu = \frac{\tilde{A}}{A}$ and the average gluon multiplicity is $\langle n_g \rangle = \mu(e^{AY}-1)$.

We have the variance as

$$D^2 = \mu e^{AY}(e^{AY}-1) \tag{43}$$

We obtain the second correlative moment to be

$$f_2 = \frac{\langle n_g \rangle^2}{\mu} \tag{44}$$

Then the normalized exclusive cross-section for producing n_g gluons is

$$\frac{\sigma^{(0,q)}_{n_g}}{\sigma_{tot}} = P_{0,1;n_g,1}(Y) = \frac{\mu(\mu+1)...(\mu+n_g-1)}{n_g!}\left[\frac{\langle n_g \rangle}{\langle n_g \rangle+\mu}\right]^{n_g}\left[\frac{\mu}{\langle n_g \rangle+\mu}\right]^{\mu} \tag{45}$$

This is a Polya-Egenberger distribution, where μ is takes half-integer values.

The corresponding generating function is

$$Q = \sum_{n_g=0}^{\infty} u_g^{n_g} u_q P_{0,1;n_g,1}(Y) = u_q\left[\frac{e^{-AY}}{1-u_g(1-e^{-AY})}\right]^{\mu} \tag{46}$$

3.4.3 Hadronization

We obtained the second correlative moment to be a positive value but experimental data show that it can take on negative values. To explain this, we add the hadronization stage to the gluon quark cascade.

We use the binomial distribution $P(X=k) = C_N^k p^k(1-p)^{N-k}$ to obtain the generating function of hadronization

$$Q^{(H)} = \sum_{k=0}^{N} P_k z^k = (1-p+pz)^N \tag{47}$$

The average multiplicity is

$$\langle k \rangle = \left.\frac{\partial Q^{(H)}}{\partial z}\right|_{z=1} = NP(1-p+pz)^{N-1} = Np \tag{48}$$

Then we find p to be

$$p = \frac{\langle k \rangle}{N} \tag{49}$$

We can then get the second correlative moment as

$$f_2 = Np(N-1)p - (Np)^2 = \frac{-\langle k \rangle^2}{N} < 0 \tag{50}$$

Combining the two stages, we get the generating function as

$$Q(z) = \sum_{m=0}^{M_g} P_m^P P_n^H \tag{51}$$

For the first stage, we have P_m^P as follows

$$P_m^P = \frac{k_p(k_p+1)...(k_p+m-1)}{m!} \left(\frac{k_p}{k_p+\overline{m}}\right)^{k_p} \left(\frac{\overline{m}}{k_p+\overline{m}}\right)^m, \tag{52}$$

where $k_p = \mu = \frac{\tilde{A}}{A}$ and $\overline{m} = \langle m_g \rangle$. The corresponding quark generating function is

$$Q^{(q)} = \left(\frac{k_p}{k_p+\overline{m}}\right)^{k_p} \left(1 - z\frac{\overline{m}}{k_p+\overline{m}}\right)^{-k_p} \tag{53}$$

The parton generating function is

$$Q^{(P)} = \left(\frac{k_p'}{k_p'+\overline{m}'}\right)^{k_p'} \left(1 - z\frac{\overline{m}'}{k_p'+\overline{m}'}\right)^{-k_p'}, \tag{54}$$

where $k_p' = 2k_p$ and $\overline{m}' = 2\overline{m}$. From now on we will drop the prime and use the convention that $\overline{m}$ is the average multiplicity for two quarks. Then

$$P_m^P = \frac{k_p(k_p+1)...(k_p+m-1)}{m!} \left(\frac{k_p}{k_p+\overline{m}}\right)^{k_p} \left(\frac{\overline{m}}{k_p+\overline{m}}\right)^m \tag{55}$$

is the multiplicity distribution for partons at the first stage.

Then from (47), (51) becomes

$$Q(z) = \sum_{m=0}^{M_g} P_m^P \left[1 + \frac{\overline{n}}{N}(z-1)\right]^{(2+\alpha m)N}, \tag{56}$$

where $\overline{n} = \overline{n}_q$ and $\alpha = \frac{\overline{n}_g}{\overline{n}_q} = \frac{N_g}{N_q}$. We can then obtain the second correlative moment as

$$f_2 = Q''(z)\Big|_{z=1} - \left(Q'(z)\Big|_{z=1}\right)^2 = \left(\frac{\alpha^2\overline{m}^2}{k_p} + \alpha^2\overline{m} - \frac{2+\alpha\overline{m}}{N}\right)(\overline{n}^h)^2 \tag{57}$$

At low energy, $\overline{m} \approx 0$, so we can neglect the terms with $\overline{m}$ and the second correlative moment becomes

$$f_2 = \frac{-2}{N}(\overline{n}^h)^2 < 0 \tag{58}$$

3.5 Gluon Decay of Bottomonium

Bottomonium particle is an upsilon meson made up of a bottom quark and antiquark. According to QCD, it can decay into 3 gluons and each gluon can produce hadrons. We can use the gluon jet and predict the multiplicity distribution of hadrons for this decay.

The multiplicity distribution for a single gluon is

$$P_m^{(g)} = \frac{1}{\overline{n}_g}\left(1 - \frac{1}{\overline{n}_g}\right)^{m-1} \tag{59}$$

The corresponding generating function is

$$G(z) = \frac{z}{\overline{m}}\left[1 - z\left(1 - \frac{1}{\overline{m}}\right)\right]^{-1} \tag{60}$$

For a three-gluon jet, the generating function is

$$G^{(3g)} = \frac{z^3}{\frac{\overline{m}^3}{3}}\left[1 - z\left(1 - \frac{1}{\frac{\overline{m}}{3}}\right)\right]^{-3}, \tag{61}$$

where $\overline{m}$ is the average multiplicity for all 3 gluons. Then the multiplicity distribution is

$$\begin{aligned} P_m &= \frac{1}{m!}\frac{\partial^m}{\partial z^m}\frac{z^3}{\left(\frac{\overline{m}}{3}\right)^3} \\ &= \frac{(m-1)(m-2)}{2\left(\frac{\overline{m}}{3}\right)^2}\left(1 - \frac{1}{\frac{\overline{m}}{3}}\right)^{m-3} \end{aligned} \tag{62}$$

We now add hadronization to the gluon cascade in the following way

$$\begin{aligned} Q(s,z) &= \sum_{m=3}^{\infty} P_m^{(g)}\left[1 - \frac{\overline{n}_g}{N_g}(1-z)\right]^{mN_g} \\ &= \sum_{m=3}^{\infty} \frac{(m-1)(m-2)}{2\left(\frac{\overline{m}}{3}\right)^2}\left(1 - \frac{1}{\frac{\overline{m}}{3}}\right)^{m-3}\left[1 - \frac{\overline{n}_g}{N_g}(1-z)\right]^{mN_g} \end{aligned} \tag{63}$$

Then the generating function for the 3-gluon is

$$Q^{(g)} = \sum P_m^{(g)} z^m = \frac{\left[1 - z\left(1 - \frac{1}{\frac{\overline{m}}{3}}\right)\right]^{-3}}{\left(\frac{\overline{m}}{3}\right)^3} \tag{64}$$

The minimum number of gluons here is equal to three, but we can have m' more gluons, making the total number of gluons $m = 3 + m'$. Then (63) becomes

$$Q(s,z) = \sum_{m'=0}^{\infty} P_{m'}^{(g)} \left[1 - \frac{\overline{n}_g}{N_g}(1-z)\right]^{(3+m')N_g} \tag{65}$$

, where

$$P_{m'}^{(g)} = \frac{(m'+2)(m'+1)}{2\left(\frac{\overline{m}}{3}\right)^2}\left(1 - \frac{1}{\frac{\overline{m}}{3}}\right)^{m-3} \tag{66}$$

Then we get the second correlative moment as

$$f_2 = \left(\frac{1}{3}\overline{m}'^2 + \overline{m}' - \frac{\overline{m}' + 3}{N_g}\right)\left(\overline{n}_g^h\right)^2 \tag{67}$$

Experimental data show that $f_2 = -2.57 < 0$ which indicates that $\overline{m} \approx 0$, i.e., we can neglect the terms with $\overline{m}$ making the second correlative moment

$$f_2 = \frac{-3}{N_g}\left(\overline{n}_g^h\right)^2 \tag{68}$$

4 Analysis of Multiplicity Distribution

We developed a C++ program that models the multiplicity distribution of charged particles produced in electron-positron annihilation into hadrons at energy of 22 GeV. We used the following data published in this [2] paper.

Charged Multiplicity (n)	Multiplicity Distribution (P_n)
2	0.1631 ± 0.0895
4	1.7797 ± 0.2557
6	7.8243 ± 0.5185
8	16.7981± 0.7497
10	22.9196 ± 0.8749
12	21.5560± 0.8322
14	14.5702± 0.6494
16	8.2160± 0.4705
18	3.6614 ± 0.2927
20	1.6538± 0.1931
22	0.5892 ± 0.1048
24	0.1637± 0.0513
26	0.0697 ± 0.0312
28	0.0355± 0.0253

Table 1: Charged Multiplicity Distribution

We found the parameters of $P_n = \Omega \sum_{m=0}^{M} P_m C^n_{(2+\alpha m)N} (\frac{\overline{n}^h}{N})^n (1 - \frac{\overline{n}^h}{N})^{(2+\alpha m)N-n}$ at 22 GeV to be as follows

k_p	3.162 ± 2.504
$\overline{m}$	2.095 ± 0.795
$\overline{n}^h$	4.651 ± 0.304
N	27.352 ± 14.204
α	0.205 ± 0.071
Ω	2.000 ± 0.0354
χ^2	1.663

Table 2: Multiplicity Distribution Parameters

We used ROOT data analysis software to draw the multiplicity distribution, see Figure 2.

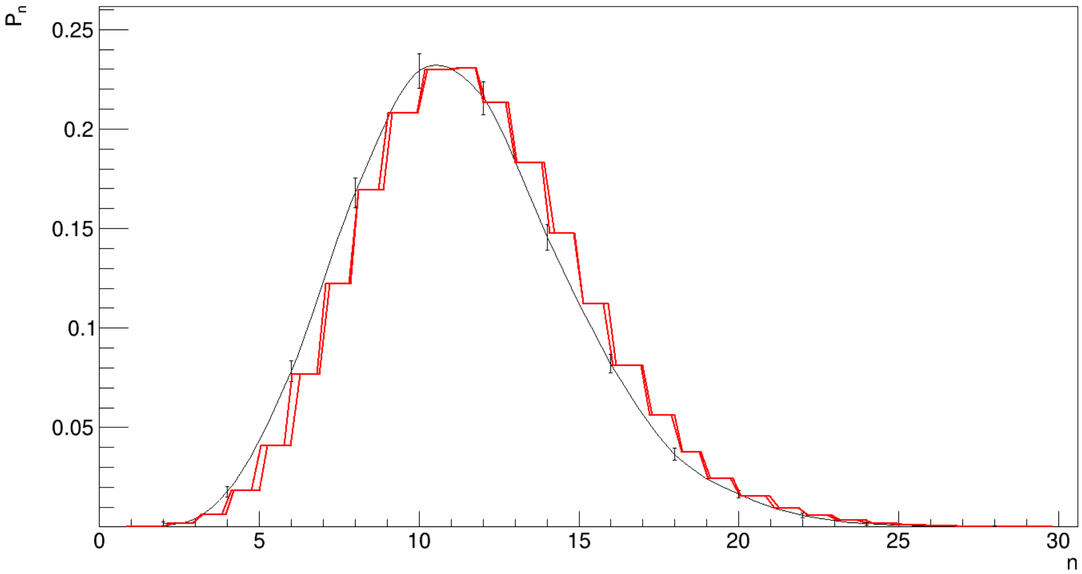

Figure 2: Multiplicity Distribution at 22 GeV

Comparing our result, Figure 2, with the one, Figure 3, published in the paper, we can see that the distributions are very similar.

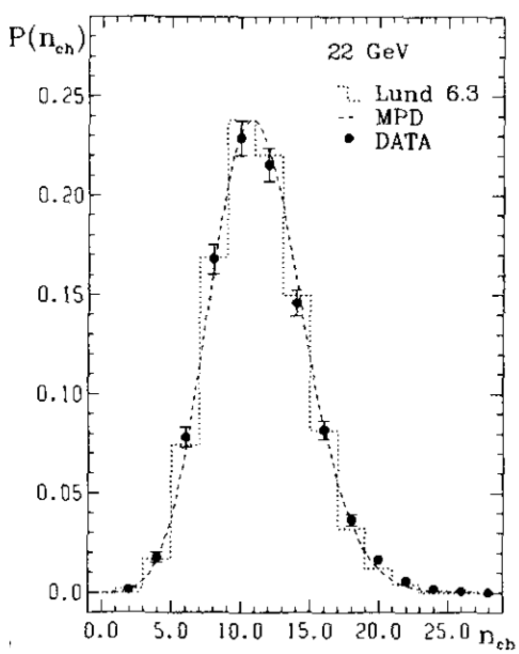

Figure 3: Multiplicity Distribution at 22 GeV

5 Conclusion

Examination of multiparticle production in high-energy physics, particularly in the context of electron-positron annihilation, has provided valuable insights into the fundamental aspects of strong interactions. The study of MP has not only led to the identification of intriguing phenomena like jets but has also presented challenges, especially at higher energy levels where inelastic channels increase, posing difficulties for conventional methods of description.

To address these challenges, our analysis employed statistical methods, with a specific focus on the Two Stage Model (TSM), which treats QCD jets as Markov branching processes. This stochastic approach allowed for a probabilistic description of parton showers within hadrons, yielding clear and comprehensive solutions for the parton multiplicity distribution.

Our study applied the TSM to calculate the multiplicity distribution for both neutral and charged particles in electron-positron annihilation at high energy. The results were extended to the decay of bottomonium into three gluons with hadronization.

Moreover, we wrote a C++ code that shows the multiplicity distribution of charged particles produced during the process of electron-positron annihilation into hadrons at an energy of 22 GeV and our findings demonstrated that the Two Stage Model agrees with experimental data on multiplicity distribution.

In summary, our investigation has contributed to the advancement of our knowledge in the realm of multiparticle production, showcasing the applicability and reliability of the Two Stage Model in describing complex processes at high energy levels. The calculated multiplicity distributions for various scenarios, along with the consistency with experimental data, underscore the significance of our approach in providing a robust framework for understanding and modeling multiparticle production in high-energy physics.

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