**JOINT INSTITUTE FOR NUCLEAR RESEARCH**

Veksler and Baldin laboratory of High Energy Physics

**FINAL REPORT ON THE INTEREST PROGRAMME**

# Puzzles of Multiplicity

**Supervisor:**

Dr. Elena Kokoulina

**Student:**

Yara Shousha, Egypt
Alexandria University

**Participation period:**

October 30 – December 10, Wave 9

## Abstract

The multiplicity distribution -number of secondaries- of charged particles produced from $e^-e^+$ annihilation is studied at energies of 14, 22, 34.8 and 43.6 GeV. We analyzed our study using the negative binomial distribution and Markov branching idea to fit them with the experimental data obtained by TASSO detector at PETRA.

The fitting is done using a CERN's ROOT minimization library. And the fitting parameters of parton stage and the stage of hadronization in this model are obtained in terms of the center of mass energies. Irregularities in the fitting parameters are found.

## Contents

**1 Introduction** **1**

1.1 Two Stage Model . . . . . . . . . . . . . . . . . . . . . . . . 1

1.2 Branching Cases . . . . . . . . . . . . . . . . . . . . . . . . 1

1.3 Markov Processes . . . . . . . . . . . . . . . . . . . . . . . . 2

1.4 Calculations . . . . . . . . . . . . . . . . . . . . . . . . . . 2

1.5 Gluon Decay Calculations . . . . . . . . . . . . . . . . . . . 3

**2 Method** **5**

**3 Results And Discussion** **6**

**4 References** **9**

**5 Acknowledgement** **10**

# 1 Introduction

The theory of strong interactions, Quantum Chromodynamics (QCD), still has a lot of unsolved problems. One of these problems is the multiparticle production (MP). Over the past few years, the MP processes have been studied to help us better understand the strong interaction and the structure of matter; through the testing of suggested phenomenological models [1]. Our work is mainly focused on the MP of the charged particles. This process starts at high energies; 14 to 189 GeV [2], where the number of secondaries increases. In accordance with QCD, they can be obtained through the production of $\gamma$ or $Z^0$–boson into two quarks:

$$e^+e^- \rightarrow (Z^0/\gamma) \rightarrow q\overline{q}$$

## 1.1 Two Stage Model

We define this process using two stages:

1. cascade stage
2. hadronization stage

Parton spectra in QCD quark and gluon jets were studied by working at the leading logarithm approximation and avoiding IR divergences by considering finite x, the probabilistic nature of the problem has been established. Giovannini proposed to interpret the natural QCD evolution parameter Y:

$$Y = \frac{1}{2\pi b} ln[1 + \alpha_s b ln(\frac{Q^2}{\mu^2})]$$

where $2\pi b = \frac{1}{6}(11N_C - 2N_f)$ for a theory with $N_C$ colours and $N_f$ flavours, as the thickness of the jets and their development as Markov process.

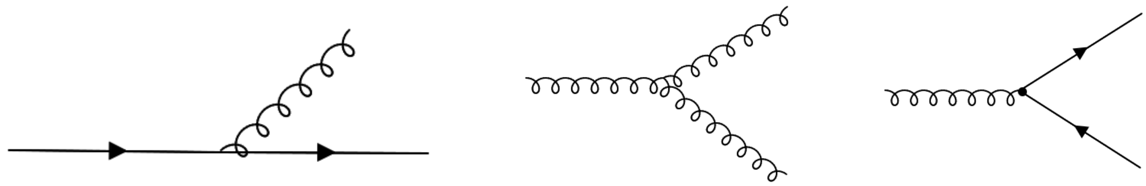
## 1.2 Branching Cases

The partons (quarks and gluons) that are produced can undergo fragmentation using one of the following ways:

1. $q \rightarrow q + g$ (Quark Bremsstrahlung)
2. $g \rightarrow g + g$ (Gluon Fission)
3. $g \rightarrow q + \overline{q}$ (Quark Pair Creation)

These three cases are shown in Figure 1. We use statistical techniques to study the branching processes of partons, because the number of particles in these processes is usually greater than 60 [3].

Let $A\Delta Y$ be the probability that a gluon in the infinitesimal interval $\Delta Y$ will convert into two gluons, $\widetilde{A}\Delta Y$ be the probability that a quark



(a) Quark Bremsstrahlung (b) Gluon Fission (c) Quark Pair Creation

Figure 1: Partons Branching Cases

will radiate a gluon, the quark continuing on its way, and $B\Delta Y$ be the probability that a quark-antiquark pair will be created from a gluon. A, $\widetilde{A}$, B to be Y-independent constants and each individual parton acts independently from others, always with the same infinitesimal probability.

### 1.3 Markov Processes

In the probability theorem, we can calculate the probability of success k-times for an event:

$$P(k) = C_n^k p^k (1-p)^{n-k} \tag{1}$$

where k is the number of success times and n is the total number of trials.

Now, we can use the idea of Giovannini [4] to describe the quark-gluon jets by Markov branching processes. Markov processes can be used when the probability of the process happening is only dependant on the previous process, but independent of all other previous processes [5].

The method of generating functions is a very important tool in the study of stochastic processes with a discrete state space.

### 1.4 Calculations

In the Markov branching processes, we define a generating function:

$$Q = \sum_{k=0}^{\infty} P_k \, z^k \tag{2}$$

Where $P_k$ is the probability of event success mentioned in equation 1, and z is an arbitrary factor.

Therefore, by substituting:

$$Q = \sum_{k=0}^{\infty} C_n^k p^k (1-p)^{n-k} \, z^k = \sum_{k=0}^{\infty} C_n^k (pz)^k (1-p)^{n-k} \tag{3}$$

Using Newton's binomial theorem $(a+b)^n = \sum_{k=0}^{\infty} C_n^k a^k b^{n-k}$:

$$Q = \sum_{k=0}^{\infty} C_n^k (pz)^k (1-p)^{n-k} \tag{4}$$

$$= [1 - p + pz]^n \tag{5}$$

$$Q = [1 + p(z-1)]^n \tag{6}$$

### 1.5 Gluon Decay Calculations

In our situation, the quarkonium decay can be represented by [6]:

$$e^- + e \to Q\widetilde{Q} \to 3 \text{ gluons} \to 3 \text{ jets}$$

We are going to focus our attention on gluons decay. In this case, p and n in equation 6 can be written as:

$$p = \frac{\overline{n}_g^h}{N_g}$$

$$n = \alpha N_g$$

Where $\overline{n}_g^h$ is the Average multiplicity of hadrons that forms from a single gluon.

And $N_g$ is the Maximum possible number of hadrons that can be produced from a single gluon.

Since, we talk here about the decay of gluons only, we write $N_g$ as N, and $\overline{n}_g^h$ as $\overline{n}^h$, just for simplicity.

So, we can substitute in equation 6:

$$Q = [1 + \frac{\overline{n}_g^h}{N_g}(z-1)]^{\alpha N_g} \tag{7}$$

Where $\alpha = \frac{\overline{n}_g^h}{\overline{n}}$ is the probability to produce a hadron from a gluon.

To obtain the generating function of hadronization for m number of gluons:

$$Q^H = [1 + \frac{\overline{n}^h}{N}]^{2N} [1 + \frac{\overline{n}^h}{N}]^{\alpha N m}$$

$$= [1 + \frac{\overline{n}^h}{N}]^{2N+\alpha N m}$$

$$= [1 + \frac{\overline{n}^h}{N}]^{(2+\alpha m)N} \tag{8}$$

The probability of producing m gluons from quark pair at a quark-gluon cascade:

$$Q(s,z) = \sum_{k=0}^{\infty} p_m [1 + \frac{\overline{n}^h}{N}]^{(2+\alpha m)N} \tag{9}$$

The Multiplicity Distribution Function ($P_n$) is the probability of obtaining a definite number of particles produced from collisions. In order to obtain multiplicity distribution for n number of particles, we take the nth derivative of the generating function with respect to the arbitrary variable z, while substituting with z = 0, and then use the following formula:

$$P_n = \frac{1}{n!} \frac{\partial^n}{\partial z^n} \, Q(s,z)|_{z=0} \tag{10}$$

$$P_n = \Omega \sum_{m=0}^{\infty} P_m \frac{(2+\alpha m)N \, [(2+\alpha m)N - 1] \, ... \, [(2+\alpha m)N - n + 1]}{n!} \times \left(\frac{\overline{n}^h}{N}\right)^n \left(1 - \frac{\overline{n}^h}{N}\right)^{(2+\alpha m)N - n} \tag{11}$$

Where $P_m$ is the negative binomial distribution function and it represents the probability that parton will be transformed into m gluons:

$$P_m = \frac{k_p(k_p+1) + ... + (k_p + m - 1)}{m!} \left(\frac{k_p}{k_p + \overline{m}}\right)^{k_p} \left(\frac{\overline{m}}{k_p + \overline{m}}\right)^m$$

This equation can also be called Polya-Egenberger distribution, with $k_p$ as a non-integer value.

## 2 Method

In our approach, we tried fitting this multiplicity distribution function and finding the 6 fitting parameters where:

- $k_p = \frac{\widetilde{A}}{A}$,
- $\overline{m}$ is the average multiplicity of gluons,
- $n_h$ is the Average multiplicity of hadrons that forms from a single gluon,
- $N$ is the Maximum possible number of hadrons that can be produced from a single gluon,
- $\alpha = \frac{\overline{n}_g^h}{\overline{n}}$ is the probability to produce a hadron from a gluon ,
- $\Omega$ is the Normalization coefficient

We used CERN's ROOT of version 6.20. "Minuit2" library which is a new object-oriented implementation of the popular FORTRAN's MINUIT minimization package was chosen for the fitting process.

We used the data from the TASSO detector at PETRA taken in 1989 at centre of mass energies ranging from 14, 22, 34.8 and 46.8 GeV [7], to compare with our phenomenological model and calculations.

TASSO detector was a part of the German national laboratory DESY. It consists of a large magnetic solenoid, 440 cm long and with a radius of 135 cm producing a field of about 0.5 Tesla parallel to the beam axis. The field of the solenoid is compensated by two coils placed symmetrically with respect to the central magnet. The solenoid is filled with tracking chambers and will be surrounded by detector elements to measure the energy and position of photons and to identify charged particles. [8]

## 3 Results And Discussion

Fitting was done using the four center of mass energies $\sqrt{s} = 14$, 22, 34.8 & 43.6 GeV, as shown in Figures 2-5. The blue line and error bars represent the experimental data and the red line represents the fitting equation. The obtained fitting parameters are shown in Table 1.

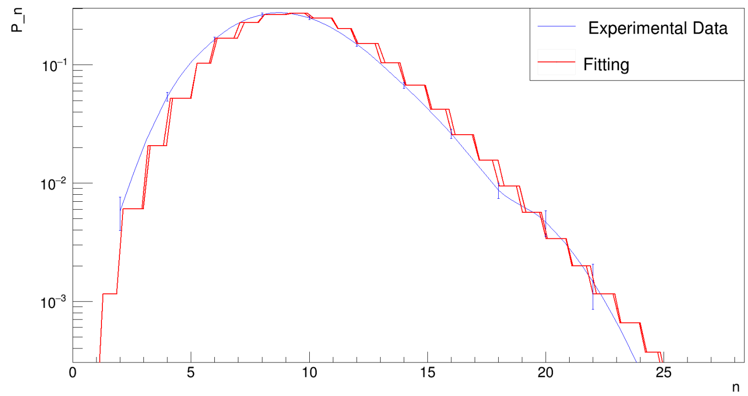



Figure 2: 14 GeV

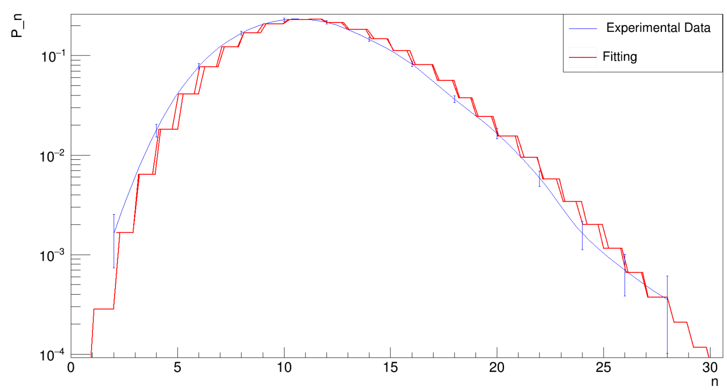



Figure 3: Multiplicity Distribution function at energy of 22 GeV

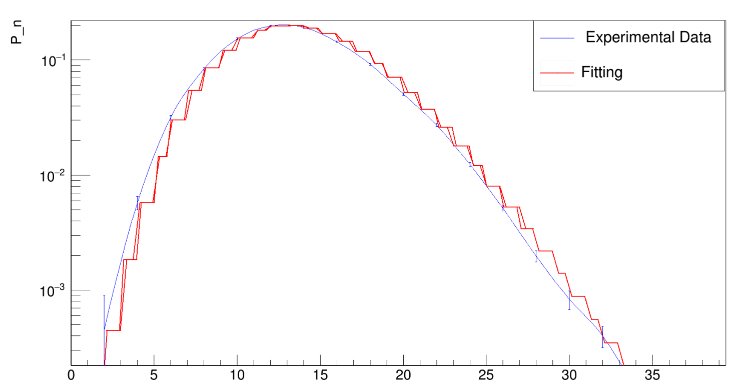



Figure 4: Multiplicity Distribution function at energy of 34.8 GeV

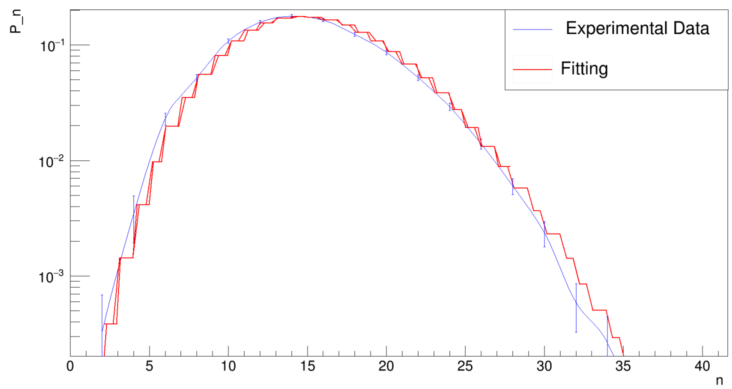



Figure 5: Multiplicity Distribution function at energy of 43.6 GeV

| | $k_p$ | $\overline{m}$ | $n^h$ | $N$ | $\alpha$ | $\Omega$ |
|---|---|---|---|---|---|---|
| $\sqrt{s} = 14\ GeV$ | 184393 | 0.0812846 | 4.46792 | 27.8873 | 0.978672 | 1.99770 |
| $\sqrt{s} = 22\ GeV$ | 3.10218 | 3.00057 | 4.53666 | 25.5509 | 0.16392 | 1.99916 |
| $\sqrt{s} = 34.8\ GeV$ | 2.14864 | 5.55918 | 5.51266 | 113.715 | 0.0836628 | 1.99886 |
| $\sqrt{s} = 43.6\ GeV$ | 582.229 | 20.4674 | 1.81107 | 35.1364 | 0.309082 | 1.99668 |

Table 1: Parameters obtained using fitting

These fitting parameters are drawn as functions of $\sqrt{s}$ in Figure 6-11. The studied model is on good agreement with the experimental data, showing no deviations in any of the curves.

But we can deduce an irregular behaviour in these parameters with respect to the energy, except for $\omega$ which shows the constancy of its value. It is also clear that there is a large error in some of the plots which is due to the extreme complexity of our equation. But the results are still acceptable.

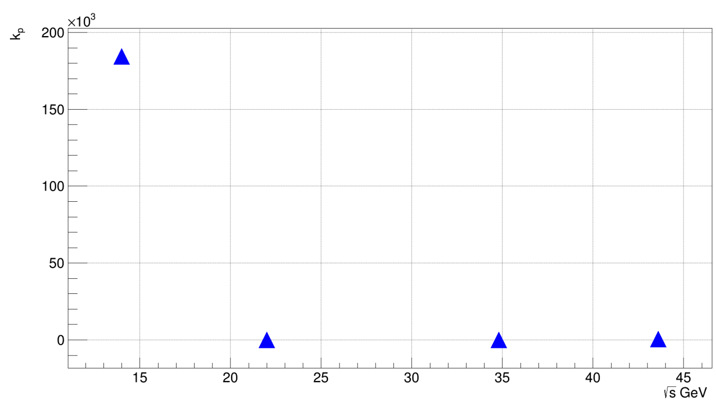



Figure 6: $k_p$

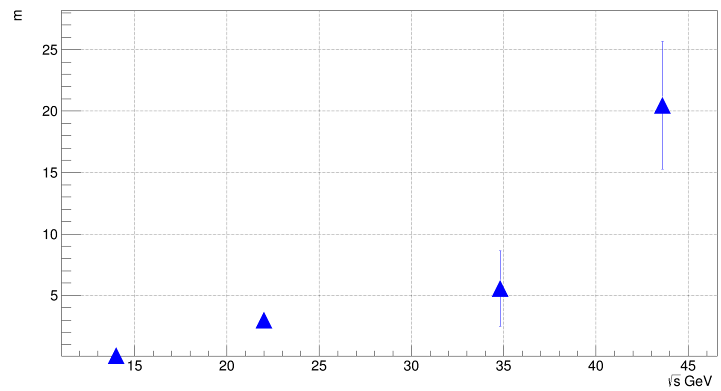



Figure 7: m

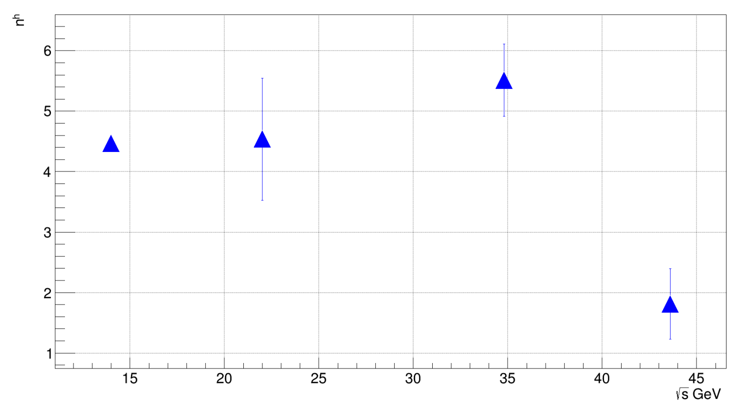



Figure 8: $n^h$

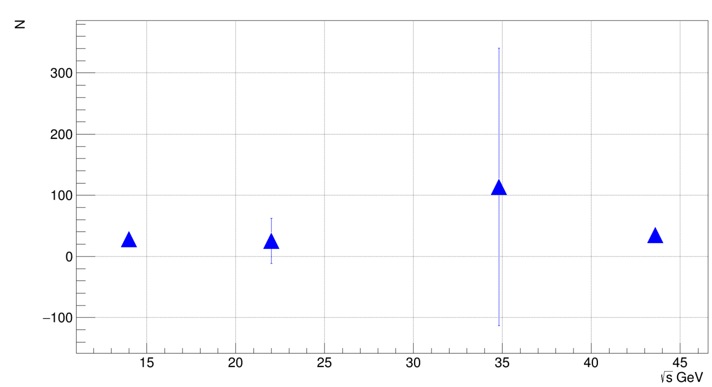



Figure 9: N

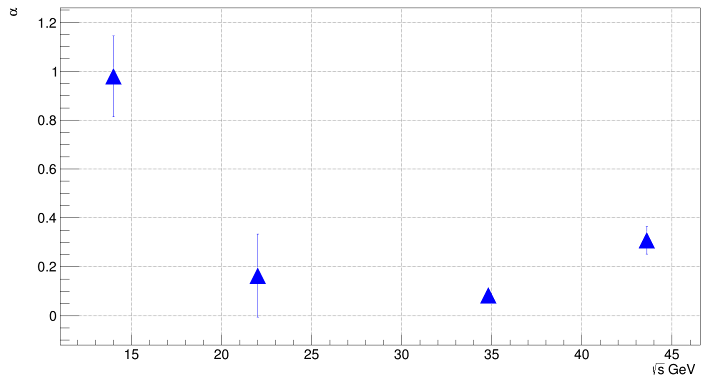



Figure 10: $\alpha$

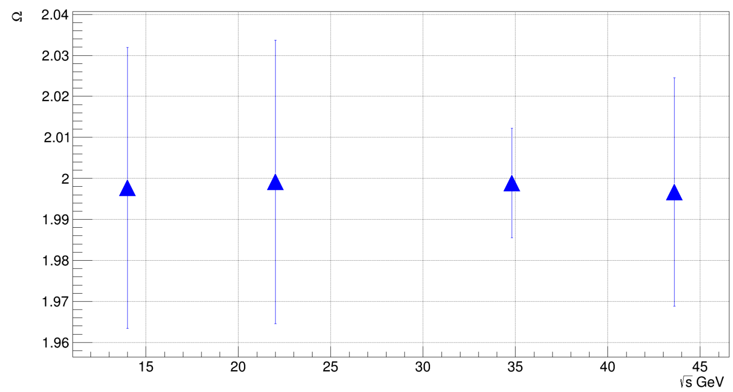



Figure 11: $\Omega$

# 4 References

[1] H David Politzer. *Asymptotic freedom: An approach to strong interactions*. 1974.

[2] E. S. Kokoulina. *Multiplicity Distribution studies of $e^+e^-$-annihilation at 50-61.4 GeV and 172-189 GeV by Two Stage Model hadronization*. 2002. arXiv: hep-ph/0209334 [hep-ph].

[3] R. M. Barnett et al. "Review of Particle Physics". In: *Phys. Rev. D* 54 (1 July 1996), pp. 1–708. DOI: 10.1103/PhysRevD.54.1. URL: https://link.aps.org/doi/10.1103/PhysRevD.54.1.

[4] A. Giovannini, S. Lupia, and R. Ugoccioni. "Multiplicity distributions in high energy collisions". In: *Nuclear Physics B - Proceedings Supplements* 25 (1992), pp. 115–123. ISSN: 0920-5632. DOI: https://doi.org/10.1016/0920-5632(92)90385-6. URL: https://www.sciencedirect.com/science/article/pii/0920563292903856.

[5] R. Syski. "Elements of the Theory of Markov Processes and Their Applications (A. T. Bharucha-Reid)". In: *SIAM Review* 5.2 (1963), pp. 173–174. DOI: 10.1137/1005059. eprint: https://doi.org/10.1137/1005059. URL: https://doi.org/10.1137/1005059.

[6] K. Koller, H. Krasemann, and T. F. Walsh. "The 3 gluon decay of quarkonium". English (US). In: *Zeitschrift für Physik C Particles and Fields* 1.1 (Mar. 1979), pp. 71–80. ISSN: 0170-9739. DOI: 10.1007/BF01450382.

[7] W. Braunschweig et al. "Charged multiplicity distributions and correlations in e+e- annihilation at PETRA energies". In: *Zeitschrift für Physik C Particles and Fields* 45.2 (June 1989), pp. 193–208. DOI: 10.1007/BF01674450. URL: https://doi.org/10.1007/BF01674450.

[8] R. Brandelik et al. "Properties of hadron final states in e+e- annihilation at 13 GeV and 17 GeV center of mass energies". In: *Physics Letters B* 83.2 (1979), pp. 261–266. ISSN: 0370-2693. DOI: https://doi.org/10.1016/0370-2693(79)90699-3. URL: https://www.sciencedirect.com/science/article/pii/0370269379906993.

## 5 Acknowledgement

I would like to express my deepest appreciation to my project supervisor, Dr. Elena Kokoulina who gave me so much time and effort throughout the International Students Practice (ISP) and helped me personally and professionally during the whole trip. And since then, she never stopped helping. During INTEREST period, she always motivates me. I'm extremely grateful to all what she has done.

I am also thankful to Krittika Sarkar, my project partner who insisted on contacting me and was so supportive and helpful. I would like to extend my sincere thanks to Ahmed ElTohamy, a colleague and a very supportive friend, with whom I took the INTEREST application journey. And luckily, we both got accepted.

I'd like to acknowledge Prof. Mohamed El-Borai who introduced me to JINR practices and helped me to apply and get accepted.