Puzzles of Multiplicity
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Abstract

The combination of the strong nuclear force and the quantum mechanics has led to the evolution of quantum chromodynamics that has played an important role in understanding the technicalities of multiparticle production. Multiparticle production helps us to expound various other kinds of reactions in terms of quarks and gluons. The same is useful in observing hadrons and generally exhibits the property of possessing a high multiplicity. All multiparticle processes can be described by a two stage model. This model will be described in this report and the calculations along with the results will be shown henceforth.

1 Introduction

Physicists always focus on accelerating of stable particles. Some of the reactions in which multiparticle production occurs are: electron-positron annihilation, proton-proton inelastic scattering, proton-antiproton annihilation, three-gluon decay quarkoniums, lepton-hadron interactions and last but not the least, collisions of high energy ions. As a matter of fact, in Fermilab, the experiment of making 900GeV protons collide into 900 GeV antiprotons confirmed the presence of top quarks.

Importance of topological cross section

A topological cross section σ_n is the cross section of creation of n charged particles at the interaction of two initial particles. Total inelastic cross section is the sum of all topological cross sections: $\sigma_{inel} = \Sigma_n \sigma_n$. The ratio of the topological cross section to inelastic cross section determines the probability of formation of n secondary particles $P_n = \sigma_n/\sigma_{inel}$.

In according to QCD the main language for description of interaction is based on quarks and gluons. How to make transit to the real observable hadrons? This theory of strong interaction (QCD) doesn't give us answer. The importance of the cross section measurement is that it gives us a tool to check the correctness of the Standard Model with addition of hadronization described by phenomenological way. Then we can compare experimental data to theoretical predictions.

2 Two stage model (TSM)

To explain the multiparticle production, a Gluon Dominance Model(GDM) has been developed, which is a convolution of two stages. The first stage is a part of QCD while the second stage, which is the hadronization stage, a phenomenological model is used. The simplest case of study combination of QCD and phenomenological scheme of hadronization is electron-positron annihilation. It will be taken into consideration in this report. It accounts for the quark-gluon cascade, causing quarks to emit gluons, gluons can give fission and eventually all quarks and gluons must turn into hadrons at the hadronization stage.

Analysing it theoretically as well as using the data available, it has been shown that explaining of the quark gluon cascade is easier with the dominance of bremsstrahlung gluons. The valence quarks stay in the leading particles and are passive, while the gluons are single-handedly responsible for the high multiplicity of the process.
In according to QCD the probabilities of three elementary processes are:

$$P(q \to qg) = C_F \frac{1+z^2}{1-z},$$

$$P(g \to gg) = 2C_A \frac{[1-z((1-z)^2)]}{z(1-z)},$$

$$P(g \to q\bar{q}) = T_F[z^2 + (1-z)^2],$$

And integrals from them are:

$$A = \int_{\epsilon'}^{1-\epsilon'} P(g \to gg)\, dz = \frac{C_A}{\epsilon}$$

$$\tilde{A} = \int_{\epsilon'}^{1-\epsilon'} P(q \to qg)\, dz = \frac{C_F}{\epsilon},$$

$$B = \int_{\epsilon'}^{1-\epsilon'} P(g \to q\bar{q}) dz = \frac{2T_F}{3}$$

$$\epsilon = (-2ln\epsilon')^{-}1$$

where z is the parton momentum fraction and ϵ' is the cut-off parameter

It's comfortable to use the generation function Q(s,z) = $\Sigma_{n=0}^{\infty} P_n z^n$. The multiplicity distributions are calculated as follows:

$$P_n(s) = \frac{1}{n!} \frac{\partial^n}{\partial z^n} Q(s.z)|_{z=0}$$

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The factorial moments are calculated as:

$$F_k(s) = \overline{n(n-1)....(n-k+1)} = \frac{\partial^k}{\partial z^k} Q(s,z)|_{z=1},$$

k=1, 2, ...
The variance of the distribution is:

$$D = \sigma^2 = \overline{n(n-1)} - \overline{n}^2 + \overline{n}$$

$$= F_2 - \overline{n}^2 + \overline{n}$$

$$= f_2 + \overline{n},$$

where $f_2 = F_2 - F_1^2$ is the second correlative moment, $F_1 = \overline{n}$ is the average multiplicity.

From Giovannini's work, [**?**] we find that the Markov nature of elementary processes such as gluon fission, quark pair-creation from gluon, quark bremsstrahlung at the first order of α_s lead to the description of various QCD jets (quark and gluon). The stochastic approach plays an important role in finding the multiplicity distribution of parton jets as a function of their thickness and elementary processes of cross sections.

The natural QCD parameter is:

$$Y = \frac{1}{2\pi b} log[1 + \alpha blog \frac{Q^2}{\mu^2}],$$

$Y = 0$ at $Q = \mu$ the first stage can start. We call Y the thickness value of a quark or a gluon which gives origin to a quark or a gluon jet.Three main elementary processes contribute with different weights to the overall quark or gluon distributions inside QCD jets:

(i) gluon fission: $g \to g + g$,
(ii) quark bremsstrahlung: $q \to q + g$,
(iii) quark pair creation: $g \to q + \bar{q}$.

The probability generating functions for a gluon jet and a quark jet will be respectively:

$$G(u_g, u_q; Y) = \sum_{n_g, n_q = 0}^{\infty} P_{1,0;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q}$$

$$Q(u_g, u_q; Y) = \sum_{n_g, n_q = 0}^{\infty} P_{0,1;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q}.$$

P_{m_g,m_q,n_g,n_q} represents the probability of m_g gluons and m_q quarks being transformed into n_g gluons and n_q quarks over a jet of thickness Y.From a probabilistic point of view the total parton population (m_g gluons and m_q quarks) evolving through thickness Y behaves as $(m_g + m_q)$ independent parton populations, each with one initial quark or gluon; this fact summarizes the branching Markov chain nature of the process.

3 Three-gluon decay of bottomium

We can apply this approach to the three-gluon decay of upsilon meson (bottommium). For this purpose it's necessary to define for three-gluon state by using a single gluon jet.

The generation function for a single gluon is given by:

$$Q(s,z) = \frac{z}{\overline{m}/3}[1 - z(1 - \frac{1}{\overline{m}/3})]^{-1}$$

$$Q^{(3g)} = [Q^{(g)}]^3 = (\frac{z}{\overline{m}/3}[1 - z(1 - \frac{1}{\overline{m}/3})]^{-1})^3$$

where $\overline{m}/3$ is the average multiplicity of every gluon jet.

$$Q^{(3g)} = \frac{z^3}{(\overline{m}/3)^3}[1 - z(1 - \frac{1}{\overline{m}/3})]^{-3}$$

$$P_m = \frac{1}{m!}\frac{\partial^m}{\partial z^m} Q^{(3g)}(z)$$

So,

$$P_m = \frac{(m-1)(m-2)}{2(\overline{m}/3)^3}(1-\frac{1}{\overline{m}/3})^{m-3}$$

According to the experimental behavior of the second correlative moment f_2 which gives the negative value we should choose for the hadronization stage multiplicity distribution with negative f_2. The most known for that distribution is the binomial one:

$$P_n = C_N^n p^n (1-p)^{N-n},$$

where n is the number of successes, N - maximum of trails. The probability of single success can be expressed as the ratio of two values: $\overline{n}^h$ and N, average number and maximum number of hadrons that can form at the second stage (hadronization).

The above equation is the expression for the binomial distribution with a second correlation moment for hadrons from the parton(quark or gluon) jet in the second stage, i.e., the hadronization stage in the Gluon Dominance Model. At energies of a few GeV when the number of partons at the stage of the qg-cascade is small, the hadronization is predominant and determines the sign of the second correlative moment that is negative.

The g-stage and hadronization is represented by:

$$Q(s,z) = \sum_{m=3}^{\infty} P_m^g [1 - \frac{\overline{n}^h}{N}(1-z)]^{mN_g}$$

The multiplicity distribution function for the two gluon decay is given by:

$$f_2 = \frac{\partial^2}{\partial z^2} Q(z) \mid_{z=1} - (\frac{\partial}{\partial z} Q(z))^2 = n(\bar{n-1}) - \bar{n}$$

Now coming to the 3-gluon decay, the multiplicity distribution of hadrons is given by the expression:

$$P_n(s) = \sum P_m C_{mN}^n (\frac{\bar{n}}{N})^n (1-\frac{\bar{n}}{N})^{mN-n}$$

$$f_2 = < n(n-1) > - < n >^2 = D_2 - < n > = < n^2 > - < n >^2 - < n >$$

The gluon distribution is:

$$\begin{aligned} P_m^{(g)} &= \frac{1}{(m-3)!}\frac{\partial^m - 3}{\partial z^{m-3}}[1 - z(1-\frac{1}{m/3})]^-3|_{z=0} \\ &= \frac{1}{(m-3)!}(1-\frac{1}{\bar{m}/3})^{m-3}\frac{1}{(\bar{m}/3)^3}\frac{(m-1)!}{2} \qquad (1) \\ &= \frac{(m-2)(m-1)}{2(\overline{m}/3)^3}(1-\frac{1}{\bar{m}/3})^{m-3}. \end{aligned}$$

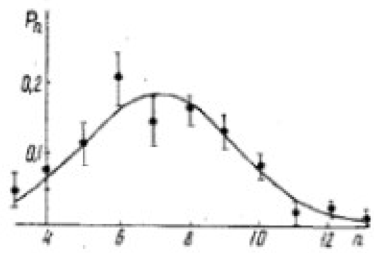

Figure 1: *three gluon decay for bottomium*

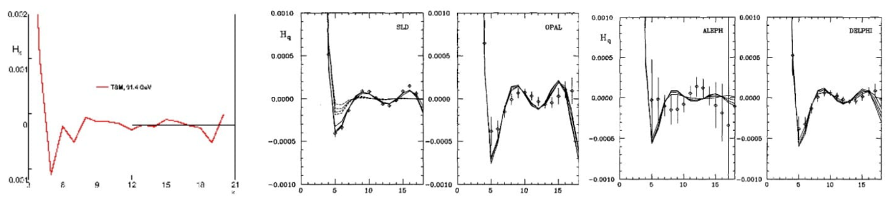

Figure 2: *factorial, cumulative moments and their ratio*

And,

$$f_2 = (\frac{\bar{m}^2}{3} - \frac{\bar{m}}{N_g} - \frac{3}{N_g})$$

Multiplicity distribution for the three gluon decay of bottomonium has the following view:

$$P_n(s) = \sum_{m'} \frac{(m'+1)(m'+2)}{2(\overline{m}/3)^2}(1-\frac{1}{\overline{m}/3})^{m'} C^{m}_{(3+m')N_g}(\frac{\overline{n}^h_g)}{N_g})^n(1-\frac{\overline{n}^h_g}{N_g})^{(3+m')N_g-n}$$

In figure 1 the multiplicity of distribution of charged particles is presented. Experimental data are shown by black points and the curve corresponds to model description.

In figure 2 we show the ratio of cumulative to factorial moments calculated by two-stage model (left figure) and four figures (on the right) from data (OPAL, DELPHI, SLD, and ALEPH collaborations).

In the conclusion, we have shown that using of the convolution of two stage of multiparticle production the three-gluon decay into hadrons can be described well.

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Photo Credits:

Both the figures have been provided by my mentor, Prof. Kokoulina which are the results of the simulation of the experimental data and curve fitting, using the CERN Root software.

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