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“Active role of gluons in hadron interactions at high multiplicity”

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Content

Introduction

In high energy physics multiparticle production (MP) is a process with producing of more than 2 particles. The main factors that characterize MP are multiplicity, topological cross section, distribution and its moments (dispersion, second correlation moment). Multiplicity is the number of secondary particles that were produced. There is another important parameter – the number of events expected in particular reaction. This value that characterizes the probability of interaction is called cross section. Topological cross describes summary cross section of all possible channels of reaction.

To investigate multiparticle processes different accelerators and detectors are used. Some information and results can be obtained from high energy cosmic rays. To describe data from carried out experiments Monte Carlo generators are built, but they are working good only at particular energies.

Modern theory of strong interaction is quantum chromodynamics (QCD) which is applicable where perturbation theory works. QCD describes many processes in the language of quarks and gluons. However, quantum chromodynamics can’t describe hadronization. Due to this fact, modeling of experiments is applied and phenomenological models are created and tested.

Gluon dominance model consists of 2 stages. The first one is cascade stage where quarks and gluons jets are described as Markov branching processes. In case of *pp*-collision there are some numbers of valence quarks with gluons, that appears as a result of melting of hadronic matter. The second one is hadronization stage. After quark-gluon cascade partons are grouping together to form observable hadrons. The number of produced hadrons that can’t be calculated with application of perturbative QCD. Model of hadronization stage is based on experimental data (e.g. $f_2<0$ at low energies but it changes sign with increasing).

e^+e^- annihilation

Distribution function

Earlier we have analyzed the process of e^+e^- annihilation which leads to birth of quarks and gluons jets.

$$e^+e^- \rightarrow \gamma\ (Z^0) \rightarrow q\bar{q} \rightarrow qg - cascade$$

In our approach the description of qg-cascade by Markov branching processes was the basic instrument. In the first stage we have taken into account 3 main elementary processes in QCD: gluon fission, quark gluon emission by a quark and quark-antiquark pair creation [5]. According to our model, the gluon jet in the stage of partons cascade fission is described by negative binomial distribution

$$P_m = \frac{\mu(\mu+1)\dots(\mu+m-1)}{m!}\left(\frac{\mu}{\mu+\bar{m}}\right)^{\mu}\left(\frac{\bar{m}}{\mu+\bar{m}}\right)^{m} \qquad (1)$$

Here μ is ratio between probabilities of process of quark bremsstrahlung and gluon fission, $\bar{m}$ is an average number of gluons produced. P_m describes the probability of producing m gluons.

The second stage – hadronization – is described with a sub narrow binomial distribution. It was chosen based on analysis of experimental data (e^+e^- annihilation at the energy lower than 9 GeV). Second correlation moment is negative at this energy. Multiparticle distribution of hadrons is given by

$$P_n^H = C_N^n\left(\frac{\bar{n}^h}{N}\right)^n\left(1-\frac{\bar{n}^h}{N}\right)^{N-n} \qquad (2)$$

where C_N^n is binomial coefficient, $\bar{n}^h$is an average number of hadrons formed from one parton and N is maximum of produced secondary particles.

Convolution of these 2 stages (cascade and hadronization) determines the multiparticle distribution for hadrons in e^+e^- annihilation. The probability to obtain certain number n of produced charged particles from m partons at the stage of hadronization is given by

$$P_n = \Omega\sum_{m=0}^{M_G}\frac{\mu(\mu+1)\dots(\mu+m-1)}{m!}\left(\frac{\mu}{\mu+\bar{m}}\right)^{\mu}\left(\frac{\bar{m}}{\mu+\bar{m}}\right)^{m}\times$$

$$\times C_{(2+\alpha m)N}^n\left(\frac{\bar{n}^h}{N}\right)^n\left(1-\frac{\bar{n}^h}{N}\right)^{(2+\alpha m)N-n} \qquad (3)$$

where $C_{(2+\alpha m)N}^n$ is binomial coefficient that equals to

$$C_{(2+\alpha m)N}^n = \frac{(2+\alpha m)N\big((2+\alpha m)N-1\big)\dots\big((2+\alpha m)N-n+1\big)}{n!}$$

and parameter α was included to distinguish hadrons produced from quark or gluon.

Fitting e^+e^-

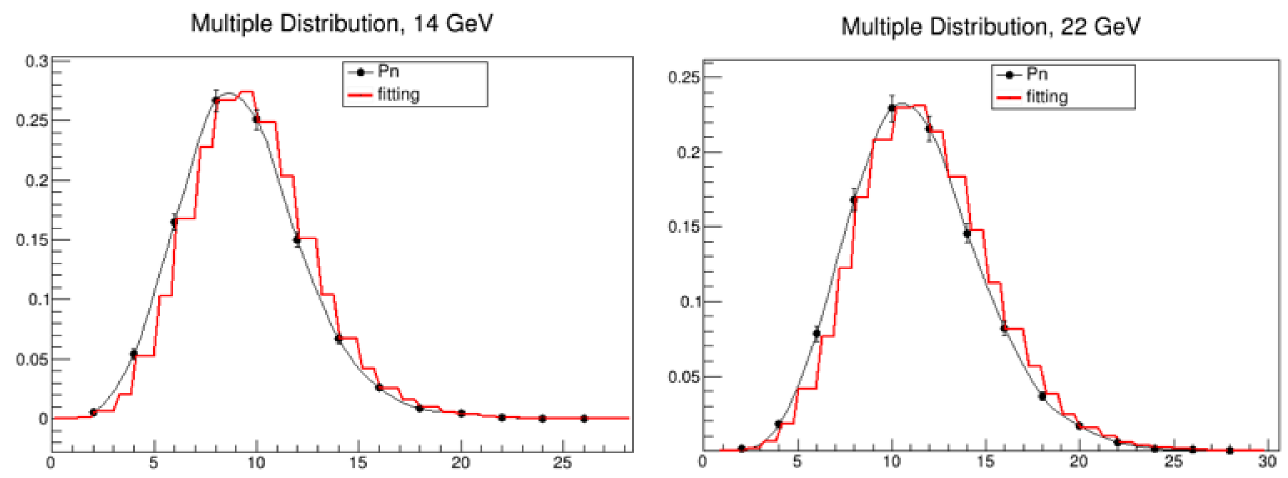
This multiplicity distribution (3) has 6 parameters:

- μ is ratio between probabilities of process of quark bremsstrahlung ($q \to q + g$) and gluon fission ($g \to g + g$)
- $\bar{m}$ is mean gluon multiplicity
- $\bar{n}^h$ is an average number of hadrons created by 1 gluon
- N is a maximum possible number of hadrons produced from 1 gluon
- α is a ratio between average numbers of gluons and quarks produced ($\bar{n}_g/\bar{n}_q$)
- Ω is the coefficient of normalization

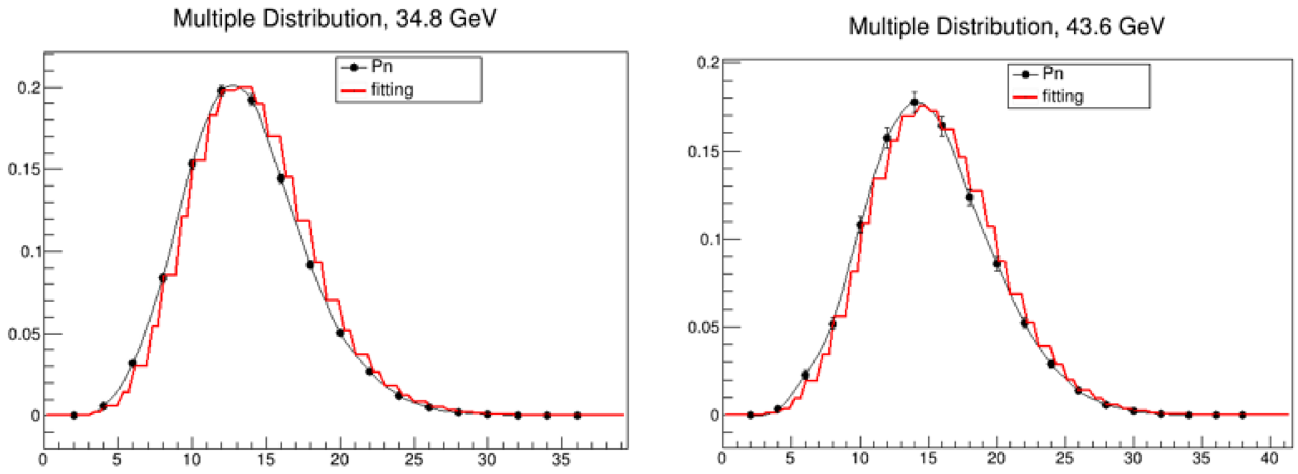
It is clear that last parameter Ω must be equal 2 because of the law of conservation of charge. Our model and calculation allow formation of any number of particle (even and odd) but in reality, appearance of odd number of particles is impossible. Before collision the system is electrically neutral – that means that during hadronization stage it shouldn't be electrically charged too. Thus, all charged particles have to born in pairs (positive and negative), so probabilities with odd-numbered n are not taken into account.

We fitted function (3) and find parameters using Fumili2 minimization package of CERN ROOT. Data were taken from [1]. Fitting was carried out for 4 energies: 14, 22, 34.8 and 43.6 GeV. Program script written in C++ is given in Application.

Results of fitting are shown in Figures 1-4. The Red line represents fitting equation and the black one is corresponding experimental data with errors. Derived parameters for every are given in the Table 1.



Figures 1-2. Distribution function for e^+e^- annihilation. The fitted function is shown in red line, the experimental data [1] are shown with black points (the curves are to guide the eye). Left: distribution for 14 GeV. Right: distribution for 22 GeV.



Figures 3-4. Distribution function for e^+e^- annihilation. The fitted function is shown in red line, the experimental data [1] are shown with black points (the curves are to guide the eye). Left: distribution for 34.8 GeV. Right: distribution for 43.6 GeV

Table 1. Results of fitting - obtained parameters for different energies.

Energy $\sqrt{s}$, GeV	14	22	34.8	43.6
μ	279219	3.174	0.6836	51.83
$\bar{m}$	0.0813	1.954	3.214	9.724
$\bar{n}^h$	4.468	4.675	6.038	2.427
N	27.85	27.8	402856	5.565
α	0.9792	0.21395	0.08254	0.4353
Ω	1.99666	1.99914	2.01421	2.00133
NDf	7	8	12	13
χ^2	2.75432	1.66254	6.22071	5.38212

pp collision

It’s possible to construct analogical model for pp collision which also leads to producing of hadrons.

$$p + p \rightarrow A + B + C \dots$$

The main feature of this process is that there are already 6 initial quarks before collision. Masses of u and d quarks that form proton is much less mass of hadron (~ 938 MeV), therefore medium is full of gluons. All these gluons can fission and make more hadrons. This approach is called Gluon Dominance Model (GDM).

The hypothesis was put forward about 2 types of hadronization: fragmentation mechanism and recombination. The first one occurs in vacuum and results in appearance of mesons. The second one consists in the following: valent quarks interact and create leading particles. Contribution of these 2 mechanisms differs depending on energy.

First results of fitting were weird because average multiplicity of hadrons was too small ($\bar{n}^h \ll 1$) if we take into account all valence quarks and some gluons. After supposing that not all valence quarks participate at this stage, hadronization parameter is growing but still remains too small. Excluding all valence quarks, $\bar{n}^h$ becomes a little bit more than 1. This transition means that hadronization mechanism changes from fragmented to recombination. The fact that all valence quarks were eliminated from interaction corresponds that protons stay as observed leading particles.

Simplified method

To relieve our calculations let’s obtain multiplicity distribution without gluon fission. In e^+e^- annihilation model the distribution function P_m of m gluons had the form of binomial distribution. For pp process we will use Poisson distribution for P_m (shows the probability that m gluons can be produced from original parton). It works this way due to the fact that the second correlation moment for experimental distribution is negative, and Poisson distribution is narrow enough to describe this. It is shown in the Figure 5.

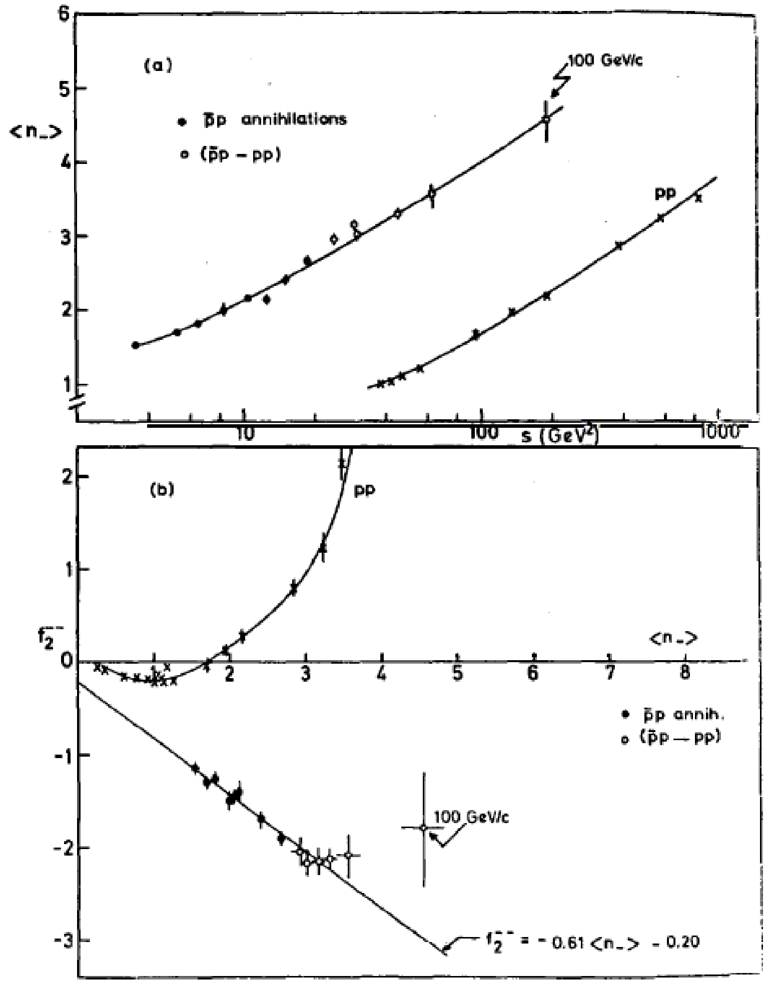


Figure 5. (a) s-dependence of mean negative multiplicity $\langle n_{-} \rangle$ for pp data (crosses). (b) plot of second correlation moment versus $\langle n_{-} \rangle$ for data as in (a). For *pp*-collision: $f_2 < 0$ at low negative multiplicity $\langle n_{-} \rangle$ (approximately s≤100 GeV^2), then it changes sign with increasing $\langle n_{-} \rangle$ and s. (ref. [4])

Therefore, multiplicity distribution function for proton-proton collision looks like

$$P_n = \Omega \sum_{m=1}^{M_G} \frac{e^{-\bar{m}}\bar{m}^m}{m!} C_{mN}^{n-2} \left(\frac{\bar{n}^h}{N}\right)^{n-2} \left(1 - \frac{\bar{n}^h}{N}\right)^{mN-(n-2)}$$

Here we've taken into account that already 2 hadrons already exist (two initial protons). Thus, the real number of formed hadrons is 2 less than we obtain.

Fitting *pp*

The obtained function has 4 parameters:

- Ω is a coefficient of normalization (must equal 2 as for e^+e^- annihilation)
- $\bar{m}$ is an average multiplicity of gluons

- $\bar{n}^h$ is an average multiplicity of hadrons produced
- N is the maximum possible number of hadrons produced from 1 gluon

We made fitting using the same instruments as in previous paragraph. Data for 100 GeV were taken from [2]. For 300 GeV results [3] was used. To get probabilities instead of cross section is possible with

$$P_n = \frac{\sigma_n}{\sum_n \sigma_n} = \frac{\sigma_n}{\sigma_{total}}$$

Results of fitting are shown in Figures 6-7. The red line represents fitting equation and black one corresponds for experimental data with errors. Modeled parameters for every energy are given in the Table 2.

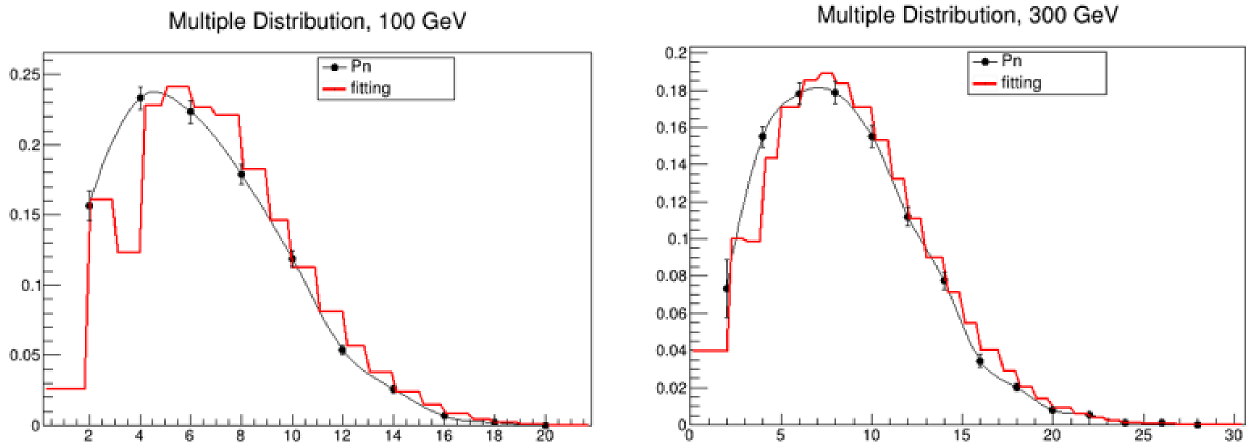


Figure 6-7. Function of multiparticle distribution for *pp* for different energies. The red line shows result of fitting, the black one represents experimental data. Left: 100 GeV with data [2], Right: 300 GeV with data [3].

Table 2. Results of fitting - obtained parameters for different energies.

Energy $\sqrt{s}$, GeV	100	300
$\bar{m}$	2.6336	3.5182
$\bar{n}^h$	1.7966	2.9169
N	3.044	6.6014
Ω	1.88329	1.7471

With gluon fission

Multiplicity distribution function cam be represented as superposition of contributions with different number of split gluons. With each subsequent gluon that experiences fission the amount of contribution decrease ($\Omega_1 \gg \Omega_2$). The probability to gain *n* hadrons equals

$$P_n = \Omega_1 \sum_{m=1}^{M_{G1}} \frac{e^{-\bar{m}}\bar{m}^m}{m!} C_{mN}^{n-2} \left(\frac{\bar{n}^h}{N}\right)^n \left(1-\frac{\bar{n}^h}{N}\right)^{mN-n} +$$

$$+ \Omega_2 \sum_{m=1}^{M_{G2}} \frac{e^{-\bar{m}}\bar{m}^m}{m!} C_{2mN}^{n} \left(\frac{\bar{n}^h}{N}\right)^n \left(1-\frac{\bar{n}^h}{N}\right)^{2mN-n}$$

where the first summand describes result of hadronization of single gluon and the second one corresponds hadronization after gluon fission.

Figure 8 shows distribution function of each summand and in total. Contribution of gluon fission plays a big role in the region of high multiplicity.

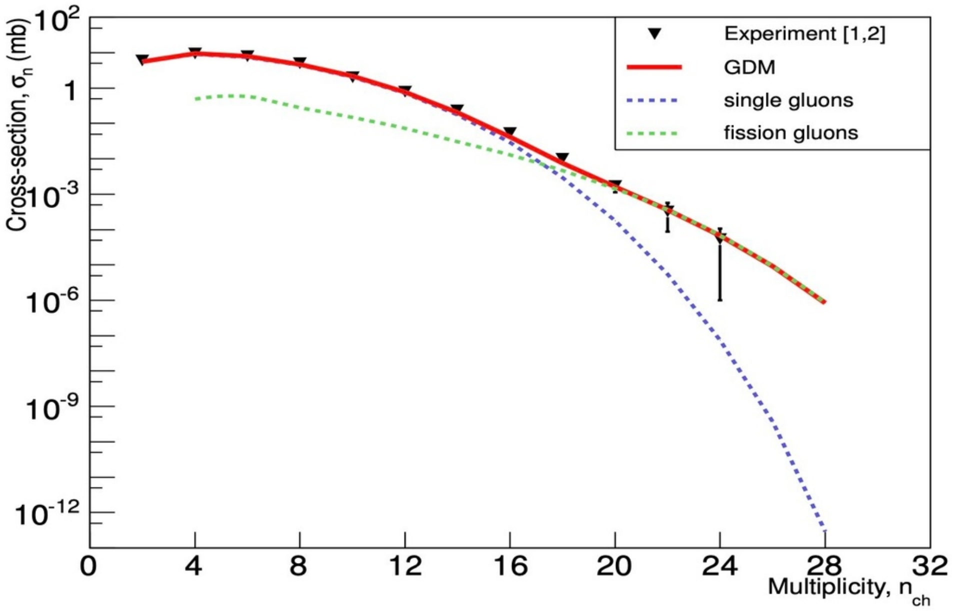


Figure 8. Experimental topological cross sections. The blue line represents contribution of gluons without fission, the green line shows role of gluons with fission. The red line is superposition of both contributions

$p\bar{p}$ annihilation

There is "quarks pairing" mechanism of $p\bar{p}$ annihilation. It's well known that proton has u, u and d quarks, while antiproton has corresponding antiquarks. The formation of three hadron jets is observed in experiment [4] in $p\bar{p}$ annihilation. In this case particles can group in pairs in different ways: to form 0 charged and 3 neutral mesons ($\bar{u}u, \bar{u}u, \bar{d}d$), 2 charged and 1 neutral mesons ($\bar{u}d, \bar{u}u, \bar{d}u$). Also, for initial valence quarks it's possible to get into pairs with quarks that randomly appear and disappear in gluon medium.

It is worth noticing that mesons are easier to form than baryons because of color theory: created hadrons must be colorless when they consist of colorful quarks. It's convenient for quarks to found one color-matching quark instead of two. Due to the fact that mesons have 2 quarks while baryons have 3 quarks in their structure, the majority of created hadrons will be mesons.

Let's identify contribution of process of 0 charged particles birth as C_0 and 2 charged particles formation as a C_2. C_4 corresponds the last-mentioned case when sea quarks appear. Multiplicity distribution should take into account contribution of every process.

$$P_n = C_0 \sum_{m=1}^{M_G} \frac{e^{-\bar{m}}\bar{m}^m}{m!} C_{mN}^n \left(\frac{\bar{n}^h}{N}\right)^n \left(1-\frac{\bar{n}^h}{N}\right)^{mN-n} +$$
$$+C_2 \sum_{m=1}^{M_G} \frac{e^{-\bar{m}}\bar{m}^m}{m!} C_{mN}^{n-2} \left(\frac{\bar{n}^h}{N}\right)^{n-2} \left(1-\frac{\bar{n}^h}{N}\right)^{mN-(n-2)} +$$
$$+C_4 \sum_{m=1}^{M_G} \frac{e^{-\bar{m}}\bar{m}^m}{m!} C_{mN}^{n-4} \left(\frac{\bar{n}^h}{N}\right)^{n-4} \left(1-\frac{\bar{n}^h}{N}\right)^{mN-(n-4)}$$

Ratio between contributions of different topologies were obtained from experimental [4]: $C_0/C_2/C_4 = 15/40/0.05$. The main share is given by process of formation 2 charged particles. Topology with all neutral particles provides less contribution, and interaction with quarks from gluon medium is almost fully suppressed by another processes.

Conclusion

For describing electron-positron annihilation a Two Stage Model was created. It is a convolution of qg-cascade, which is calculated with the help of QCD and theory of Markov branching processes, and hadronization stage, that was proposed for particular experimental data. In case of *pp*-collision there are 6 valence quarks and few gluons that affect further formation of hadrons. GDM takes into account these initial partons and gives us opportunity to get parameters of hadronization stage. There is simplified method of calculation without gluon fission, another one that includes it and their superposition. $p\bar{p}$ annihilation can be described through quark pairing into leading particles (pions). Every topology has different contribution in multiplicity distribution of observable hadrons.

Sources

[1] Charged multiplicity distribution and correlations in e^+e^- annihilation at PETRA energies (TASSO Collaboration, 1989) // Z.Phys.C – Particles and Fields 45, 193-208

[2] $\pi^+ p$, $K^+ p$ and pp topological cross sections and inclusive interactions at 100 GeV using a hybrid bubble-chamber —spark-chamber system and a tagged beam (W. M. Morse, V. E. Barnes, D. D. Carmony, R. S. Christian, A. F. Garfinkel, and L. K. Rangan; A. R. Erwin, E. H. Harvey, R. J. Loveless, and M. A. Thompson; 1977) // Physical Review D; Volume 15, Number 1

[3] pp interactions at 300 GeV/c: Measurement of the charged-particle multiplicity and the total and elastic cross section (A. Firestone, V. Davidson, D. Lam, F. Nagy„C. Peck, and A. Sheng; F. T. Dao, R. Hanft, J. Lach, E. Malamud, and F. Nezrick; 1974) // Physical Review D; Volume 10, Number 7

[4] High energy antiparticle-particle reaction differences and annihilations (Rushbrooke J.G. and Webber B.R.) // Physics Reports, V.44, No. 1., 1–92.

[5] QCD jets as Markov branching processes (A. Giovannini, 1979) // Nuclear Physics B161 429-448

[6] Particle Detectors, Second edition (Claus Grupen and Boris Shwarts, 2008) // Cambrige monographs on particle physics, nuclear physics and cosmology

[7] STUDY OF MULTIPARTICLE PRODUCTION BY GLUONDOMINANCE MODEL (E.S. Kokoulina, V.A. Nikitin) // arViv.org/abs/hep-ph/0502224v1 24 Feb 2005

Application

```
/**********************************************************************
 *                                                                    *
 * Copyright (c) 2005 ROOT Foundation,  CERN/PH-SFT                   *
 *
                                           *
 **********************************************************************/
#include <iostream>
#include "TH1.h"
#include "TF1.h"
#include "TCanvas.h"
#include "TSystem.h"
#include "TRandom3.h"
#include "TMath.h"
#include "TGraphErrors.h"
#include "Math/MinimizerOptions.h"

Double_t fun(Double_t *x, Double_t *par) {

  Int_t MG = 20;
  Int_t n, k;
 Double_t K1, K2, K3, K4;
 Double_t N1, N2, N3, N4, N5, N6, N;
 n = x[0];

 Double_t S, K, C, Sum;
 Double_t P0, PN, Pn;

 K1=par[0]+par[1];   // kp+m_
 K2=par[0]/K1;       // kp/(kp+m_)
 K3=par[1]/K1;       // m_/(kp+m_)
 K4=TMath::Power(K2, par[0]);     // (kp/(kp+m_))^kp

 N1=par[2]/par[3];   // nh/N
 N2=1.-N1;            // 1-nh/N
 N3=N1/N2;           // (nh/N)/(1-nh/N)
 N5=TMath::Power(N2,2.*par[3]);      // (1-nh/N)^2N

 S=1.;
 // m=0
 for (int i=0; i<n; i++)
   {S=S*(2.*par[3]-i)*N3/(i+1.);}  // ((nh/N)/(1-nh/N))^n*2N(2N-1)...(2N-n+1)/n!
 P0=par[5]*K4*S*N5;


 Sum=0.;
 // m=1,2...
 for (int m=1; m<=MG; m++)
   {
     K=1.;
     for (int l=1; l<=m; l++)
       {K=K*(par[0]+l-1.)*K3/l;}     // (m_/(kp+m_))^m*kp(kp+1)...(kp+m-1)/m!
     C=1.;
     for (int p=1; p<=n; p++)
       {C=C*N3*((2.+par[4]*m)*par[3]-p+1.)/p;}   // (2+am)N((2+am)N-1)...((2+am)N-
n+1)/n!*((nh/N)/(1-nh/N))^n

   N=TMath::Power(N2, (2.+par[4]*m)*par[3]);  //  (1-nh/N)^(2+am)N
   Sum=Sum+C*K*N;
   }
```

```
 PN=par[5]*K4*Sum;

 Pn=PN+P0;
return Pn ;
}

void ee22() {
  const Int_t npar = 6;

    TF1 *f1 = new TF1("f1",fun,2,28,6);
   f1->SetParameter(0,4.91);
   f1->SetParameter(1,3.01);
   f1->SetParameter(2,4.34);
   f1->SetParameter(3,10.2);
   f1->SetParameter(4,0.2);
   f1->SetParameter(5,2.0);

   Double_t xvalues1[14] = {2., 4., 6., 8., 10., 12., 14., 16., 18., 20., 22., 24., 26.,
28.};
   Double_t yvalues1[14] = {0.1631, 1.7797, 7.8243, 16.7981, 22.9196, 21.5560, 14.5702,
                            8.2160, 3.6614, 1.6538, 0.5892, 0.1637, 0.0697, 0.0355};
   Double_t evalues1[14] = {0.0895, 0.2557, 0.5185, 0.7497, 0.8749, 0.8332, 0.6494,
                       0.4705, 0.2927, 0.1931, 0.1048, 0.0513, 0.0312, 0.0253};
   for (int k=0; k<14; k++)

     {
       yvalues1[k] = yvalues1[k]/100.;
       evalues1[k] = evalues1[k]/100.;
       }

   TGraphErrors *gr1 = new TGraphErrors(14, xvalues1, yvalues1, 0, evalues1);
      ROOT::Math::MinimizerOptions::SetDefaultMinimizer("Fumili2");
      //    ROOT::Math::MinimizerOptions::SetDefaultMinimizer("Minuit2");
    gr1->SetTitle("Multiple Distribution, 22 GeV");
   gr1->Fit("f1");
   gr1->Draw("ACP");
   gr1->SetLineWidth(1);
   gr1->SetMarkerStyle(20);
   gr1->SetMarkerSize(1);
   gr1->SetLineColor(1);
   f1->SetLineWidth(3);
   f1->SetLineColor(2);
   TLegend *leg = new TLegend(0.5,0.8,0.7,0.89);
      leg->AddEntry(gr1,"Pn");
      leg->AddEntry(f1,"fitting");
      leg->Draw();
  }
```