JOINT INSTITUTE FOR NUCLEAR RESEARCH

Veksler and Baldin laboratory of High Energy Physics

FINAL REPORT ON THE INTEREST PROGRAMME

Puzzles of Multiplicity

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Contents

Abstract

Multiparticle production (MP) in high-energy physics is a key area of study that sheds light on strong interactions and the nature of matter. Processes involving the creation of multiple particles at high energy levels, particularly hadrons, offer valuable insights into fundamental physics. The investigation of MP has led to the discovery of jets, which are observable phenomena in processes generating energetic partons like electron-positron annihilation. Analyzing MP at higher energy levels presents challenges due to increased inelastic channels, requiring the use of statistical methods for accurate descriptions. By treating QCD jets as Markov branching processes, researchers can probabilistically model parton showers within hadrons, providing clear solutions for parton multiplicity distribution. This approach is applied to analyze charged particle multiplicity distributions in electron-positron annihilation experiments, revealing irregularities in fitting parameters. The evolution of quantum chromodynamics through the combination of strong nuclear force and quantum mechanics has been instrumental in understanding multiparticle production and its implications for particle physics.

Introduction

One of the key parameters observed during the experiment is multiplicity - the number of secondary particles that arise from the interaction of elementary particles. An important aspect is the multiplicity of charged particles, which plays a significant role in data analysis. Additionally, experimenters also assess the multiplicity of neutral particles, which provides a more complete picture of the processes taking place.

One of the main statistical characteristics of multiplicity is its mean value, which reflects the typical number of secondary particles in a given process. Another important parameter is the dispersion of multiplicity, which describes the degree of spread of values around the mean. Analyzing these parameters provides information about the dynamics and properties of particle interactions.

In the framework of Quantum Chromodynamics (QCD), it is possible to calculate hard processes of particle interactions at the amplitude level. However, difficulties arise when describing the hadronization stage, when quarks and gluons confine into hadrons. To account for this phase, a two-stage model is proposed, which adds a phenomenological hadronization stage to pQCD calculations.

The essence of the two-stage model is that after the hard scattering of particles in the first stage, a hadronization process occurs, during which quarks and gluons form hadrons - stable particles observable in the detector. This model allows for the inclusion of confinement effects and describes the multiplicity distribution for various processes, such as electron-positron annihilation.

Studies of particle multiplicity are crucial for understanding the physics of elementary particles and their interaction properties. Analyzing multiplicity parameters allows for testing theoretical models and comparing them with experimental data. Thus, studying multiplicity plays a key role in modern high-energy physics and helps expand our knowledge of the world at the smallest level.

Two Stage Model

The two-stage process described in the text begins with the annihilation of electron-positron pairs at high energies. This process consists of two main stages: the cascade stage and the hadronization stage. Detailed model schematic is as follows

$$e^- e^+ \to \gamma(Z^0) \to q\overline{q} \ \to (q, \overline{q}\, g) \to h'_s.$$

Broadly speaking, the process can in principle be subdivided into four steps as is illustrated in Fig. 1 [3]:

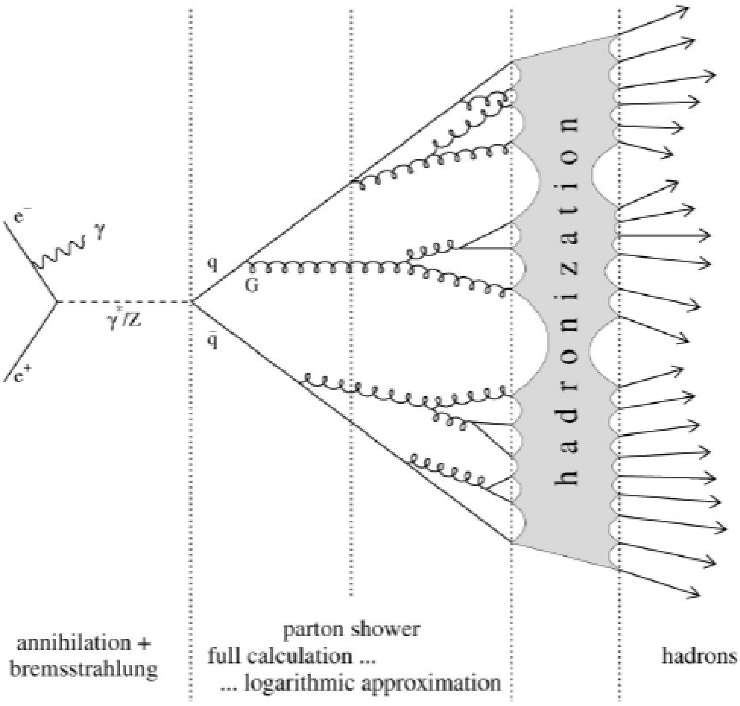

Fig.1. Schematic representation of an $e^{-}e^{+}$ annihilation process into hadrons.

In the first stage, called the cascade stage, the electron and positron annihilate into a virtual photon or Z^0 boson. These virtual particles then decay into pairs of fermions and antifermions, including quarks and antiquarks. As a result, quark and antiquark at high energies constantly bremsstrahlung, and then gluons experience fission resulting in the development of the quark-gluon cascade.

In the second stage, known as the hadronization stage, the particles initially with high energy gradually lose it and form hadrons, which are observable. Hadronization describes the conversion of quarks and gluons into hadrons. It is important to note that unlike the first stage, the hadronization stage cannot be described by perturbative quantum chromodynamics (QCD) due to the low energies of the particles involved.

Thus, the two-stage process of electron-positron pair annihilation represents a sequential development of events, starting with the high-energy splitting of particles in the first stage and ending with the formation of observable hadrons in the second stage. Each of these stages plays a key role in understanding the mechanisms of particle multiplicity production and is a subject of further research in elementary particle physics.

Cascade stage

The idea of applying branching Markov processes to Quantum Chromodynamics (QCD) in Giovannini's work [1] is that the evolution of jets formed in high-energy particle collisions can be described as a sequence of random branchings that follow certain probability laws.

Branching Markov processes are a convenient mathematical tool for modeling particle branchings in QCD because they allow describing the probabilities of different branchings and their consequences based on the current state of the system without the need to consider the entire previous history of the process.

Three elementary processes contribute into QCD jets:

(1) gluon fission;

(2) quark bremsstrahlung;

(3) quark pair creation.

Let $A\Delta Y$ be the probability that gluon in the infinitesimal interval ΔY will convert into two gluons, $\tilde{A}\Delta Y$ be the probability that quark will radiate a gluon. We can say, that process of quark pair creation is absent,because the development of this process requires large energies.

The probability that a parton will be transformed into m gluons in a jet of thickness thickness Y and call it $P_m^P(Y)$ (Y - evolution parameter).The probability generating function for a parton jet will be

$$Q^P(z;Y) = \sum_{m=0}^{\infty} P_m^P(Y) z^m. \tag{1}$$

For quark jet explicit solutions are given [1]

$$P_0(Y) = e^{-\tilde{A}Y}, \tag{2}$$

$$P_m(Y) = \frac{\mu(\mu+1)...(\mu+m-1)}{m!} e^{-\tilde{A}Y}(1-e^{-\tilde{A}Y})^m. \tag{3}$$

This formula is Polya-Egenberger distribution, where is non-integer. Then the normalized exclusive cross-section for producing m gluons is

$$\frac{\sigma_m^q}{\sigma_{tot}} = P_m(Y) = \frac{\mu(\mu+1)...(\mu+m-1)}{m!} \left[\frac{\bar{m}}{\bar{m}+\mu}\right]^m \left[\frac{\mu}{\bar{m}+\mu}\right]^\mu, \tag{4}$$

where $\mu = \dfrac{\tilde{A}}{A}$ and $\bar{m} = \mu(e^{AY}-1)$. The generating function will be given by

$$Q^q(z;Y) = \sum_{m=0}^{\infty} z^m P_m(Y) = \left[\frac{e^{-AY}}{1-z(1-e^{-AY})}\right] = \left[\frac{\mu}{\mu+\bar{m}(1-z)}\right]. \tag{5}$$

Hadronization

By combining the two stages, we obtain the generating function as

$$P_n(s) = \sum_m P_m^P P_n^H(m,s), \tag{6}$$

where P_m^P is MD for partons (4), $P_n^H(m,s)$ - MD for hadrons produced from m partons on the stage of hadronization.

According to the TSM, the stage of hard fission of partons is characterized by a negative binomial distribution (NBD) for quark jet.

$$P_m^P(s) = \frac{k_p(k_p+1)...(k_p+m-1)}{m!}\left[\frac{\bar{m}}{\bar{m}+k_p}\right]^m \left[\frac{k_p}{\bar{m}+k_p}\right]^{k_p}, \tag{7}$$

where $k_p = \dfrac{\tilde{A}}{A}$, $\bar{m} = \sum_m mP_m^P$. Parameters kp and $\bar{m}$ of MD for two joint quark-antiguark jets are doubled, but we use that designations.

P_m^P and generating function for MD $Q^P(s,z)$ are

$$P_m^P = \frac{1}{m!}\frac{\partial^m}{\partial z^m}Q^P(s,z)|_{z=0}, \tag{8}$$

$$Q_m^P(s,z) = \left[1 + \frac{\bar{m}}{k_p}(1-z)\right]^{-k_p}. \tag{9}$$

MD of hadrons formed from parton are described

$$P_n^H = C_{Np}^n \left(\frac{\bar{n}_p^h}{N_p}\right)^n \left(1 - \frac{\bar{n}_p^h}{N_p}\right)^{N_p-n}, \tag{10}$$

with generating function

$$Q_p^H = \left[1 + \frac{\bar{n}_p^h}{N_p}(z-1)\right]^{N_p}, \tag{11}$$

where n_p^h and $N_p(p = q, g)$ have meaning of average multiplicity and maximum secondaries of hadrons are formed from parton on the stage of hadronization.

MD of hadrons in e^+e annihilation are determined by convolution of two stages

$$P_n(s) = \sum_{m=0}^{\infty} P_m^P \frac{\partial^n}{\partial z^n}(Q^H)^{2+m}|_{z=0}, \tag{12}$$

where 2 is two quarks and m is gluons .

Next, we simplify the second stage by approximating $\dfrac{\bar{n}_q^h}{N_q} \approx \dfrac{\bar{n}_g^h}{N_g}$, assuming that the probabilities of hadron formation from a quark or gluon are equal. We introduce the parameter $\alpha = \dfrac{N_g}{N_q}$ to differentiate between hadron jets created from quarks or gluons in the second stage. We also simplify by setting $N = N_q$and $\bar{n}^h = \bar{n}_q^h$. This yields

$$Q_q^H = \left(1 + \frac{\bar{n}_p^h}{N_p}(z-1)\right)^N, \tag{13}$$

$$Q_g^H = \left(1 + \frac{\bar{n}_p^h}{N_p}(z-1)\right)^{\alpha N}. \tag{14}$$

By substituting equations (7) and (11) into equation (12) and taking the derivative with respect to z, we derive the MD of hadrons produced in the e^+e^- annihilation process within the TSM framework. To facilitate comparison with experimental results, a normalization factor Ω was introduced in equation, and the summation over the number of gluons was limited to M_g, which represents the maximum number of gluons that can be generated in the initial stage

$$P_n(s) = \Omega \sum_{m=0}^{M_g} P_m^P C_{(2+\alpha m)N}^n \left(\frac{\bar{n}^h}{N}\right)^n \left(1 - \frac{\bar{n}^h}{N}\right)^{(2+\alpha m)N-n}. \tag{15}$$

Method

We used root cern for fitting experimental data. The data on multiplicity distribution were taken from the TASSO detector at the PETRA accelerator, obtained in 1989 at center-of-mass energies ranging from 14, 22, 34.8 and 46.8 GeV [2].

Now let us analyze the expression by which we fit the experimental data

$$P_n(s) = \Omega \sum_{m=0}^{M_g} P_m C^n_{(2+\alpha m)N} \left(\frac{\bar{n}^h}{N}\right)^n \left(1 - \frac{\bar{n}^h}{N}\right)^{(2+\alpha m)N-n}, \tag{16}$$

where

$$P_m = \begin{cases} \left(\dfrac{k_p}{k_p+\bar{m}}\right)^{k_p}, & \text{если } m = 0; \\ \dfrac{k_p(k_p+1)...(k_p+m-1)}{m!} \left(\dfrac{k_p}{k_p+\bar{m}}\right)^{k_p} \left(\dfrac{\bar{m}}{k_p+\bar{m}}\right)^m, & \text{если } m > 0; \end{cases} \tag{17}$$

The multiplicity distribution means the probability of obtaining a definite number of particles produced from collisions, it contains information about particle correlations.

Now let us turn to the physical meaning of the six parameters, which we obtain as a result of fitting:

- Ω is normalization factor. In our case it should be equal to about 2, since the experimental data are presented only for even n,

- $k_p = \frac{\tilde{A}}{A}$ is parameter showing how the quark bremsstrahlung process and the gluon fission process are related. We expect that the process of bremsstrahlung emission will prevail,

- $\bar{m}$ is the average multiplicity of gluons before hadronization,

- $\bar{n}^h$ is the average multiplicity of hadrons, which is born from one gluon,

- N is the maximum possible number of hadrons that can be produced from a single gluon,

- $\alpha = \dfrac{\bar{n}^h_g}{\bar{n}^h_q} \approx \dfrac{\bar{N}_g}{\bar{N}_q}$ is parameter entered so that there are fewer unknown parameters.

Results And Discussion

Table 1 presents the parameter values and their uncertainties for energies 14, 22, 34.8, and 43.6 GeV.

Таблица 0.1: Parameters and their errors obtained as a result of fitting

$\sqrt{s}$, GeV	Ω	k_p	$\bar{m}$	$\bar{n}^h$	N	α	χ^2
14	1.998	16.000	0.084	4.465	27.724	0.965	2.799
	0.034	2.074	0.064	0.095	10.242	0.236	
22	1.999	3.170	1.959	4.675	27.799	0.214	1.663
	0.035	2.628	0.759	0.302	14.646	0.075	
34.8	1.998	7.530	11.148	3.972	15.000	0.128	8.849
	0.015	1.762	5.456	0.238	5.466	0.050	
43.6	2.004	40.006	34.925	1.174	7.021	0.311	5.865
	0.029	9.593	2.822	0.170	2.587	0.051	

It is interesting that the mean value of hadrons, which is born from one gluon $\bar{n}^h$ is close from 1 to 5 for all energies, it can indicate that the hadronization mechanism follows the fragmentation model. According to this model hadrons are formed as a result of breaking of parton jets.

Now let us present you the Multiplicity Distribution graphs for different energies.

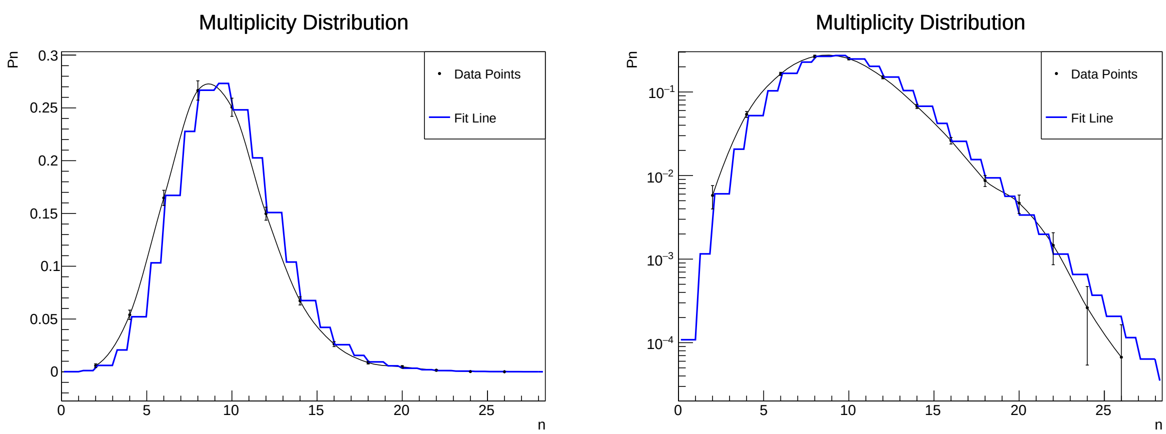

Fig.2. Multiplicity Distribution at 14 GeV.

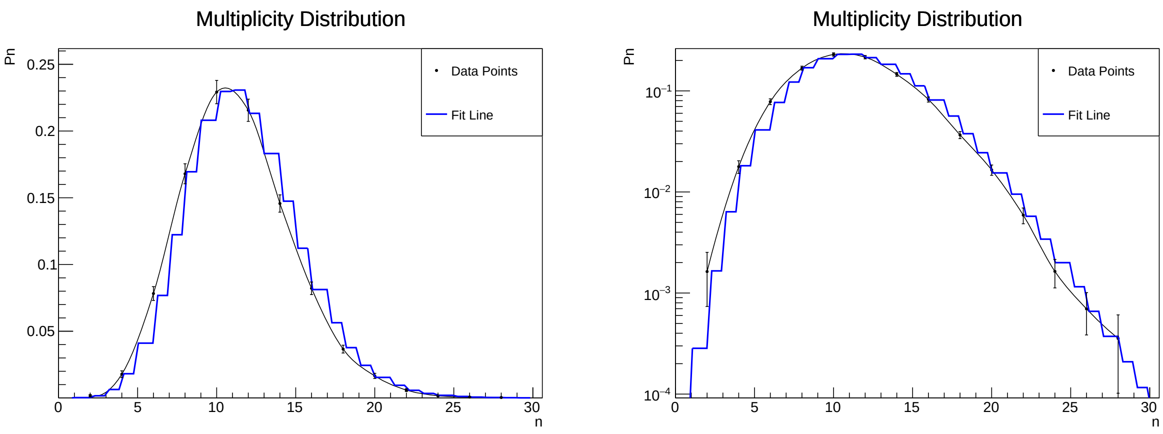

Fig.3. Multiplicity Distribution at 22 GeV.

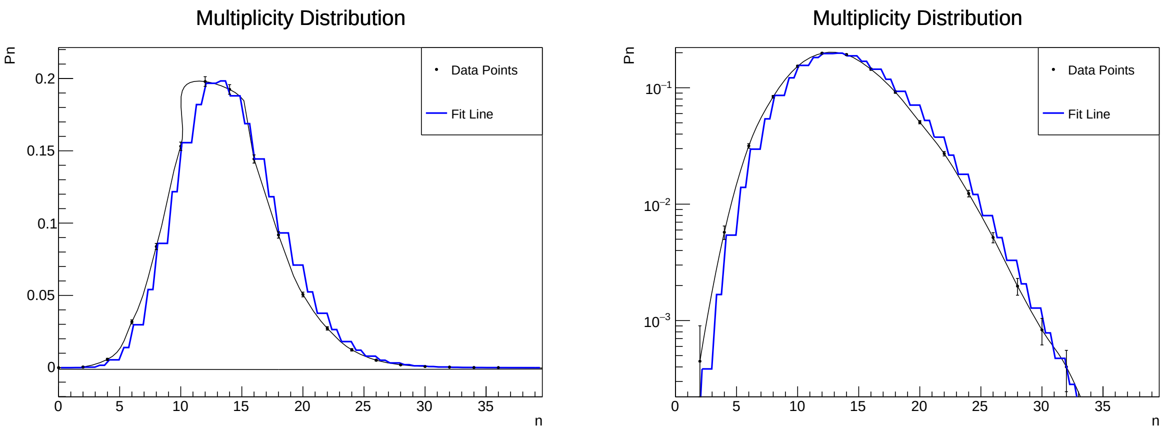

Fig.4. Multiplicity Distribution at 34.8 GeV.

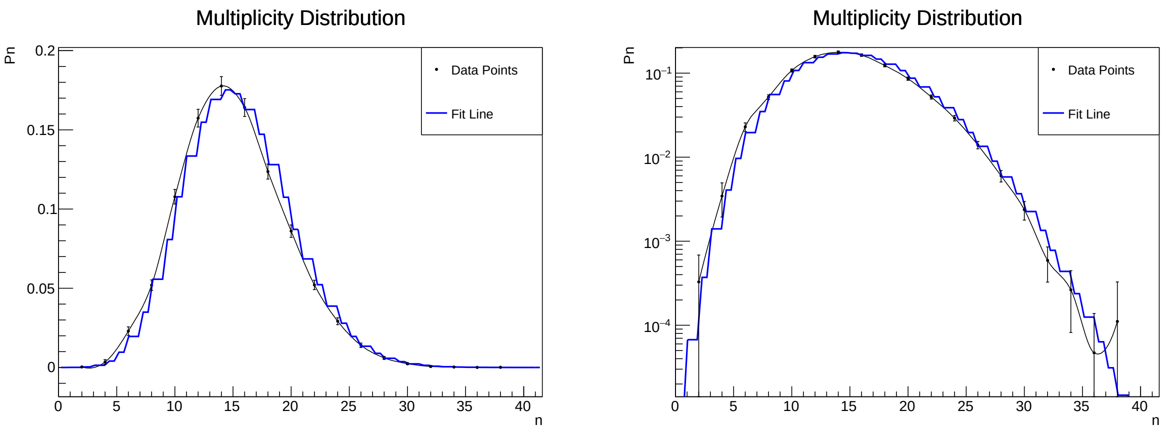

Fig.5. Multiplicity Distribution at 43.6 GeV.

Conclusion

In the experiment, one of the key parameters observed is multiplicity, which refers to the number of secondary particles generated. While pQCD enables the calculation of hard processes in particle interactions, it encounters challenges in describing the hadronization stage, where quarks and gluons combine to form hadrons. To address this issue, a two-stage model has been proposed, involving the addition of a phenomenological hadronization stage to the calculations. This model facilitates the computation of multiplicity distribution, particularly for processes such as electron-positron annihilation. Using, the two-stage model, expression (16) and (17) were obtained. Multiplicity Distribution was fit and the values of 6 parameters for energies 14, 22, 34.6, and 43.6 GeV were obtained.

Program listing

Program for analysis of events with energy of $\sqrt{s} = 14$ GeV.

```
#include <iostream>
#include <TCanvas.h>
#include <TGraphErrors.h>
#include "TMath.h"
#include <cmath>

Double_t MyFunction(Double_t *x, Double_t *par)
     {
       Double_t sum = 0;
       Int_t n = x[0];
       for (int m = 0; m < 8; m++)
        {
        Double_t Pm = 0;
        Double_t A = 1.0, B = 1.0, C = 1.0;
        if (m == 0)
           {
           Pm = TMath::Power(par[1]/(par[1]+par[2]),par[1]);
           }
           else
           {
            Double_t Pm1 = 0 ;
            Pm1 = (1 / TMath::Factorial(m)) *
            TMath::Power(par[1]/(par[1]+par[2]), par[1]) *
            TMath::Power((par[2] / (par[1]+par[2])), m );
            Double_t Pm2 = 1;
            for (int i = 1; i <= m; i++)
              {
               Double_t Pm2_2 = 0;
               Pm2_2 = par[1]+i-1;
               Pm2 *= Pm2_2;
              }
            Pm = Pm1*Pm2;
           }

                 for (int k = 1; k <= n; k++)
                     {
                         Double_t A_1 = (2+par[5]*m)*par[4]-k+1;
                         A *= A_1;
```

```
                }
                A *= (1 / TMath::Factorial(n));
                B = TMath::Power(par[3]/par[4],n);
                C = TMath::Power( 1 - par[3]/par[4],
                (2+par[5]*m)*par[4]-n);
                sum += Pm*A*B*C;

            }
            Double_t Pn_val = par[0]*sum;
            return Pn_val;

        }


int main14() {
    Double_t n[13] = {2, 4, 6, 8, 10, 12, 14,
                      16, 18, 20, 22, 24, 26};
    Double_t Pn[13] = {0.5783, 5.3986, 16.4745, 26.6517,
                       25.0602, 14.9795, 6.7216, 2.6178,
                0.8703, 0.4682, 0.1463, 0.0262, 0.0067};
    Double_t PnError[13] = {0.18, 0.4449, 0.7230, 0.9146,
                            0.8596, 0.6175, 0.3885, 0.2336,
                   0.1306, 0.1159, 0.0606, 0.0208, 0.0097};

    for (int i = 0; i < 13; ++i)
    {
        Pn[i] *= 0.01;
        PnError[i] *= 0.01;
    }

    TCanvas *c1 = new TCanvas("c1",
    "Multiplicity␣Distribution", 800, 600);
    TF1 *f1 = new TF1("f1",MyFunction,2,26,6);
    f1->SetParameter(0, 2);
    f1->SetParLimits(1, 12, 16);
    f1->SetParameter(2, 0.08);
    f1->SetParameter(3, 2.87);
    f1->SetParameter(4, 27.7);
    f1->SetParameter(5, 1);
    f1->SetLineColor(kBlue);
    f1->SetParNames("omega", "Kp", "m", "n_h", "N", "alpha");
```

```
    TGraphErrors *graph = new TGraphErrors(13,
    n, Pn, nullptr, PnError);
    ROOT::Math::MinimizerOptions::SetDefaultMinimizer("Minuit2")
    graph->Fit("f1");
    graph->SetTitle("Multiplicity Distribution; n; Pn");
    graph->SetMarkerStyle(20);
    graph->SetMarkerSize(0.5);
    graph->Draw("ACP");
    auto legend1 = new TLegend(0.7, 0.7, 0.9, 0.9);
    legend1->AddEntry( graph, "Data Points", "p");
    legend1->AddEntry(f1, "Fit Line", "l");
    legend1->Draw();
    c1->SaveAs("mMultiplicity Distribution 14.pdf");
    c1->Draw();

    TCanvas *c2 = new TCanvas("c2",
    "Multiplicity Distribution log", 800, 600);
    TGraphErrors *graph_log = new TGraphErrors(13,
    n, Pn, nullptr, PnError);
    ROOT::Math::MinimizerOptions::SetDefaultMinimizer("Minuit2")
    c2->SetLogy();
    graph_log->Fit("f1");
    graph_log->SetTitle("Multiplicity Distribution; n; Pn");
    graph_log->SetMarkerStyle(20);
    graph_log->SetMarkerSize(0.5);
    graph_log->Draw("ACP");
    auto legend = new TLegend(0.7, 0.7, 0.9, 0.9);
    legend->AddEntry( graph_log, "Data Points", "p");
    legend->AddEntry(f1, "Fit Line", "l");
    legend->Draw();
    c2->SaveAs("Multiplicity Distribution log 14.pdf");
    c2->Draw();

    return 0;
}
```

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