Puzzles of Multiplicity

Report for International Remote Student Training at JINR

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Abstract

This essay investigates the parton multiplicity distribution in quark and gluon jets, exploring the fundamental properties and characteristics of these jets in high-energy particle physics to provide insights into the internal structure and fragmentation patterns of jets. We begin by establishing the theoretical framework for understanding parton multiplicity distribution, discussing the underlying physics processes. We then examine experimental results from electron-positron annihilation experiments, fitting the multiplicity distributions to our model. Furthermore, the essay discusses the factorization of parton and hadronic phases using a Two Stage Model. It highlights the importance of this model in describing the transition from partonic to hadronic states within jets and its impact on parton multiplicity distribution. In conclusion, this essay provides a comprehensive overview of the parton multiplicity distribution in quark and gluon jets, shedding light on the intricate nature of these fundamental particles and their interactions through a theoretical construction.

Introduction

Quark and Gluon jets

Jets are robust tools for studying short-distance collisions involving quarks and gluons. With a suitable jet definition, one can connect jet measurements made on clusters of hadrons to perturbative calculations made on clusters of partons. More ambitiously, one can try to tag jets with a suitably-defined flavor label, thereby enhancing the fraction of, say, quark-tagged jets over gluon-tagged jets. This is relevant for searches for physics beyond the standard model, where signals of interest are often dominated by quarks while the corresponding backgrounds are dominated by gluons. A wide variety of quark/gluon discriminants have been proposed and there is a growing catalog of quark/gluon studies.

Due to the complicated radiative showering and fundamentally non-perturbative hadronization that occurs in the course of jets emerging from partons, there is no unambiguous definition of “quark” or “gluon” jets at the hadron-level. Despite this challenge, the importance of a clear, well-defined, and practical definition of quark and gluon jets at modern colliders cannot be overstated.

Quark and Gluon Jet Definition. *A phase space region (as defined by an unambiguous hadronic fiducial cross section measurement) that yields an enriched sample of quarks (as interpreted by some suitable, though fundamentally ambiguous criterion).*

Large Electron-Positron Collider (LEP) measured the ratio of the number of particles in gluon vs quark jets. The average multiplicity of any type of particle, along with its variance are given by the semi-classical approximation:

$$\frac{\langle N_g \rangle}{\langle N_q \rangle} = \frac{C_A}{C_f}$$

$$\frac{{\sigma_g}^2}{{\sigma_q}^2} = \frac{C_A}{C_f}$$

Where C_F and C_A are the color charges for gluons and quarks respectively, and $\frac{C_A}{C_f} = \frac{9}{4}$.

An intuitive explanation for these results is that a quark jet is dominated by the first gluon emission, at which point it continues to shower like a gluon jet. Since gluon jets have more particles, for a given energy they will have correspondingly fewer hard particles. In cases where QCD estimates do not agree with full simulation or with data, the reason is often attributed to energy conservation not being taken into account in each splitting. Since shower Monte Carlos enforce this energy conservation, they often have better agreement with data than the analytic estimates. Multiplicities have been calculated, including energy-momentum conservation, at N3LO. At LEP I energies, the result was $\frac{\langle N_g \rangle}{\langle N_q \rangle} \approx 1.7$ studied the charged particle multiplicity in light quark jets of average energy 45.6 GeV and gluon jets of 41.8 GeV. Agreement in the moments (mean, width, skewness, kurtosis) of the particle-count distributions was found to agree with the Monte Carlo event generators and with analytic predictions.

Subjet multiplicities were also examined at LEP for various subjet sizes. Extremely small subjets ($K_T = 0.1$ GeV) approach the limit of particles, and therefore probed hadronization. But larger subjets ($K_T = 5$ GeV) probed the better modeled, perturbative physics and gave the largest ratio between quark and gluon subjet multiplicities. For the first study cited, the average energy of the quark jets was 32 GeV, while that for gluon jets was 28 GeV.

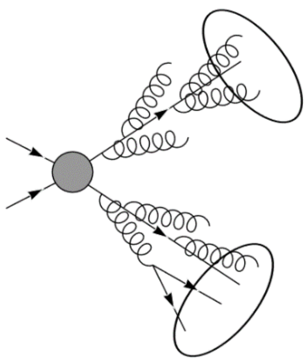


Figure 1: Dijet from initial quarks and gluon radiation.

Branching Markov Processes

A stochastic process is a counterpart of the deterministic process. Even if the initial condition is known, there are many possibilities how the process might go, described by probability distributions. More formally, a stochastic process is a collection of random variables {X(t), t ∈ T} defined on a common probability space indexed by the index set T which describes the evolution of some system. One of the basic types of stochastic process is a Markov process. The Markov process has the property that conditional on the history up to the present, the probabilistic structure of the future does not depend on the whole history but only on the present. The future is, thus, conditionally independent of the past. The original purpose of branching processes was to serve as a mathematical model of a population in which each individual in generation produces some random number of individuals in generation, according, in the simplest case, to a fixed probability distribution that does not vary from individual to individual.

Generating Function

The probability generating function of a discrete random variable is a power series representation (the generating function) of the probability mass function of the random variable. Probability generating functions are often employed for their succinct description of the sequence of probabilities Pr(*X* = *i*) in the probability mass function for a random variable *X*, and to make available the well-developed theory of power series with non-negative coefficients.

A generating function is just a different way of writing a sequence of numbers. The interest of this notation is that certain natural operations on generating functions lead to powerful methods for dealing with recurrences on the coefficients.

Definition: Let $(a_n)_{n\geq 0}$ be a sequence of numbers. The generating function associated to this sequence is the series

$$G(x) = \sum_{n\geq 0} a_n x^n$$

where a_n is the number of objects of size n in the class.

If $x = (x_1, \dots, x_d) = X$ is a discrete random variable taking values in the *d*-dimensional non-negative, the probability generating function of X is defined as:

$$G(z) = G(z_1, \dots, z_d) = \sum_{x_1,\dots,x_d}^{\infty} p(x_1, \dots, x_d) z^{x_1} \dots z^{x_d}$$

The <u>expectation</u> of X is

$$\langle x \rangle = \frac{\partial G(z)}{\partial z}\bigg|_{z=1}$$

The <u>variance</u> of X is

$$\mathrm{D}_2 = \frac{\partial^2 G(z)}{\partial^2 z}\bigg|_{z=1} - \left[\frac{\partial G(z)}{\partial z}\bigg|_{z=1}\right]^2 + \frac{\partial G(z)}{\partial z}\bigg|_{z=1}$$

And it's second correlative moment is

$$f_2 = D_2 - \frac{\partial G(z)}{\partial z}\bigg|_{z=1}$$

Two Stage Model

It was interpreted the natural QCD evolution parameter as the thickness of the QCD jets.

$$Y = \frac{1}{2\pi b} \log\left[1 + \alpha b \; log \frac{Q^2}{\mu^2}\right]$$

Where $2\pi b = \frac{1}{6}[11N_c - 2N_f]$ for a theory N_c colors and N_f flavors.

Three main elementary processes contribute with different weights to the overall quark or gluon distributions inside QCD jets:

1. gluon fission: $g \to g + g$
2. quark bremsstrahlung: $q \to q + g$
3. quark pair creation: $g \to q + \bar{q}$

Let $A\Delta Y$ be the probability that a gluon in the infinitesimal interval ΔY will convert into two gluons, $\tilde{A}\Delta Y$ the probability that a quark will radiate a gluon, the quark continuing on its way, and $B\Delta Y$ the probability that a quark-antiquark pair will be created from a gluon. $A, \tilde{A}, B$ are assumed to be Y-independent constants and each individual parton acts independently from the others.

The generating functions for a gluon jet and a quark jet will be, respectively

$$G(u_g, u_q; Y) = \sum_{n_g, n_q = 0}^{\infty} P_{1,0,n_g,n_q}(Y)\, u_g{}^{n_g} u_q{}^{n_q}$$

$$Q(u_g, u_q; Y) = \sum_{n_g, n_q = 0}^{\infty} P_{0,1,n_g,n_q}(Y)\, u_g{}^{n_g} u_q{}^{n_q}$$

Where $P_{m_g,m_q,n_g,n_q}(Y)$ is the probability that m_g gluons and m_q quarks will be transformed into n_g gluons and $n_q$quarks.

The probability for a gluon or a quark to produce, in the interval $(Y + \Delta Y)$, n_g gluons and n_q quarks through processes 1-3 under the requirement of probability conservation is, for a gluon jet:

$$P_{1,0,n_g,n_q}(Y) = \left[1 - \tilde{A} n_q \Delta Y - A n_g \Delta Y - B n_g \Delta Y\right] P_{1,0,n_g,n_q}(Y) + \tilde{A} n_q \Delta Y P_{1,0,n_g-1,n_q}(Y) \\ + A(n_g - 1)\Delta Y P_{1,0,n_g-1,n_q}(Y) + B(n_g + 1)\Delta Y P_{1,0,n_g+1,n_q-2}(Y) + o(\Delta Y)$$

And for a quark jet we only need to write $P_{0,1,n_g,n_q}$ instead of $P_{1,0,n_g,n_q}$ in every case.

The evolution of generation function for quarks and gluons can be obtained from the following system of equations:

$$\frac{\partial G}{\partial Y} = AG^2 - AG - BG$$

$$\frac{\partial Q}{\partial Y} = -\widetilde{AQ} + \tilde{A} QG$$

With initial conditions:

$$G(u_g, u_q, 0) = u_g$$

$$Q(u_g, u_q, 0) = u_q$$

Solutions in particular cases

To find the explicit solution of the cross section is not easy. However, approximate solutions can be obtained for particular cases, which can help us to understand the general problem.

The case to consider is $B = 0,\ A \neq \tilde{A} \neq 0$, meaning that no pair creation is allowed. The evolution of generation function is:

$$\frac{\partial G}{\partial Y} = AG^2 - AG$$

$$\frac{\partial Q}{\partial Y} = -\widetilde{AQ} + \tilde{A}QG$$

With initial conditions:

$$G(u_g, u_q, 0) = u_g$$

$$Q(u_g, u_q, 0) = u_q$$

Then, the probability for a gluon to produce n_g gluons is:

$$P_{1,0,n_g,0}(Y) = e^{-AY}(1 - e^{AY})^{n_g - 1}$$

And the mean number of gluons in the jet is:

$$\langle n_g \rangle = e^{AY}$$

The corresponding generating function is:

$$G = \sum_{n_g=0}^{\infty} u_g^{n_g} P_{1,0,n_g,0}(Y) = \frac{u_g e^{-AY}}{1 - u_g(1 - e^{-AY})}$$

$$D^2 = \langle {n_g}^2 \rangle - \langle n_g \rangle^2 = e^{AY}(e^{AY} - 1)$$

The probability for a quark to produce n_q quarks is:

$$P_{0,1,n_g,1}(Y) = \frac{\mu(\mu + 1) \ldots (\mu + n_g - 1)}{n_g!} e^{-AY}(1 - e^{-AY})^{n_g}$$

$$\mu = \frac{\tilde{A}}{A}$$

$$\langle n_g \rangle = \mu(e^{AY} - 1)$$

The corresponding generating function is:

$$Q = \sum_{n_g=0}^{\infty} u_g^{n_g} u_q P_{0,1,n_g,1}(Y) = u_g \left[\frac{e^{-AY}}{1-u_g(1-e^{-AY})}\right]^{\mu}$$

$$D^2 = \langle {n_g}^2 \rangle - \langle n_g \rangle^2 = \mu e^{AY}(e^{AY}-1)$$

Hadronization

Under the Two Stage Model a hadronization phase is added following a binomial distribution based on experimental fits for e^+e^- annihilation at 9 GeV which presents a negative second correlative moment at low energies. The full probability can be factorized as:

$$P_n^P(s) = \sum_m P_m^P P_n^H(m,s)$$

Where P_m^P is for partons, P_n^H is for hadrons produced from m partons and $\sqrt{s}$ is the center of mass energy.

The hadronic probability is described by:

$$P_n^H = C_{N_p}^m \left(\frac{\overline{n_p^h}}{N_p}\right)^n \left(1-\frac{\overline{n_p^h}}{N_p}\right)^{N_p-n}$$

Being $C_{N_p}^m$ the binomial coefficient.

The hadronic generating function is then:

$$Q_n^H(z) = \left[1+\frac{\overline{n_p^h}}{N_p}(z-1)\right]^{N_p}$$

Finally, the simplification $\frac{\overline{n_g^h}}{N_g} \approx \frac{\overline{n_q^h}}{N_q}$ means that hadron formation from quarks and gluons have equal probabilities.

Denoting $\alpha = \frac{N_g}{N_q}$; $N = N_q$; $\overline{n^h} = \overline{n_q^h}$; $\overline{m} = \mu(e^{AY}-1)$; $\mu = k_p$ we get

$$P_m^P = \frac{k_p(k_p+1)\ldots(k_p+m-1)}{m!}\left[\frac{\overline{m}}{\overline{m}+k_p}\right]^m \left[\frac{k_p}{\overline{k_p+m}}\right]^{k_p}$$

$$P_n(s) = \Omega \sum_{m=0}^{M_g} P_m^P C_{2+\alpha m}^m \left(\frac{\overline{n^h}}{N}\right)^n \left(1 - \frac{\overline{n^h}}{N}\right)^{(2+\alpha m)N-n}$$

Where N_g and N_q are the maximum number of hadrons produced of gluon or quark respectively, $\overline{n_g{}^h}$ and $\overline{n_q{}^h}$ are the mean value of hadrons produced of gluon and quark respectively, m is the parton multiplicity and Ω is a normalization coefficient.

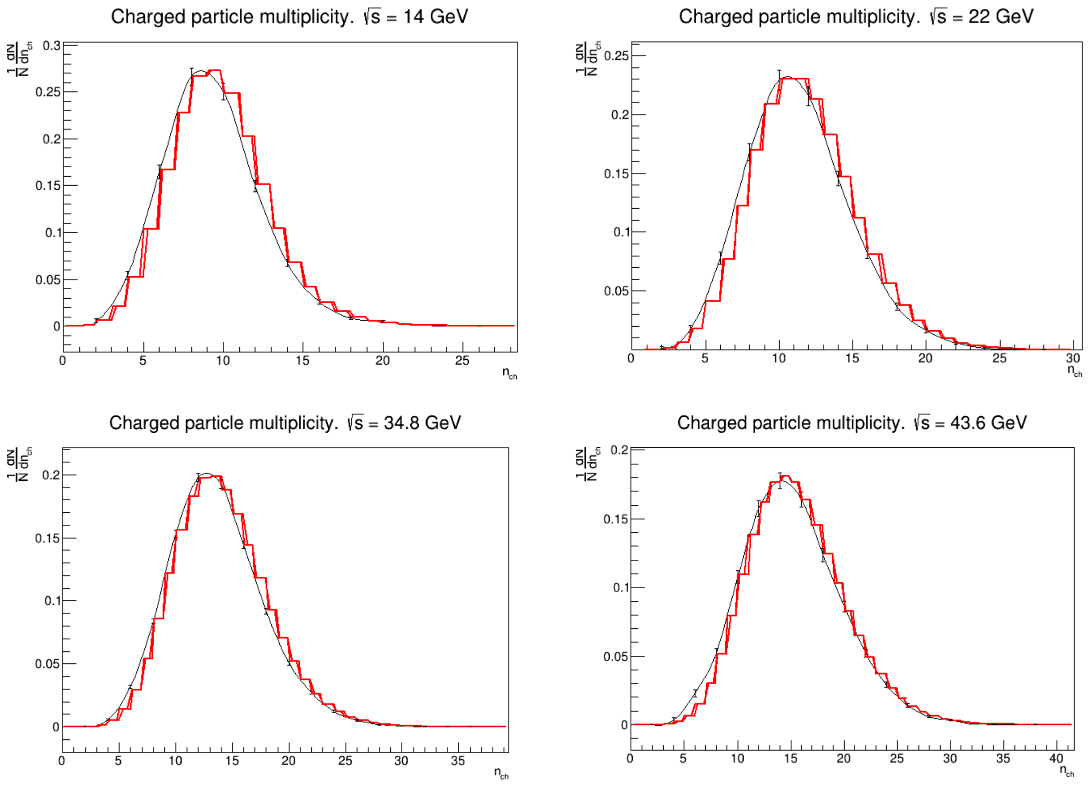
Results

The final distribution can be fitted to data for e^+e^- annihilation at different energies setting a cut on $M_g = 20$. The parameters obtained for every energy are listed in the table below, in addition, the value of χ^2 is also reported.

Table 1: Fitting parameter results at different center of mass energies.

$\sqrt{s}$ GeV	k_p	$\bar{m}$	$\bar{n}^h$	N	α	Ω	χ^2
14	20.0	0.083	4.465	27.738	0.967	1.997	2.79
22	10.0	0.85	4.859	28.0	0.383	1.998	1.72
34.8	10.0	2.0	5.213	28.0	0.303	1.997	9.16
43.6	20.0	2.0	5.6	28.0	0.348	1.987	22.87

In general, there is good agreement of data with the shape of the proposed distribution as shown in the following plots for every energy studied. It's worth noting that for the normalization coefficient ($\boldsymbol{\Omega}$) we get a consistent result across energies, the same happens for the maximum number of hadrons ($\boldsymbol{N}$) and the mean number of hadrons ($\bar{\boldsymbol{n}}^{\boldsymbol{h}}$). Analyzing the value of χ^2 for every fit, we observe greatest deviations at the highest energy despite having the grater statistics across all studied energies, also with the greatest number of degrees of freedom $NDF = 13$. Modifying the cut in maximum parton multiplicity does not alter the results significatively.



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