The Puzzle of Multiplicity: A Markovian Branching Processes Approach to QCD Jets

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1 Introduction

This essay aims to investigate the Parton multiplicity distribution in quark and gluon jets, in order to understand their internal structure and fragmentation patterns.

During a relativistic heavy-ion collision (RHIC for short), one of the possible observables is the multiplicity, the number of secondary particles that arise from the interaction of elementary particles. We will discuss that both the multiplicity of charged particles such as electrons, and of neutral charges such as the Π^0, play a significant role in data analysis. One of the main statistical characteristics of multiplicity is its mean value, the most common number of secondary particles in a given process. From this quantity, we can define the degree of spread of values around the mean, the multiplicity dispersion.

Naturally, the framework of this investigation is QCD, however, it's very hard to describe the hadronization stage, when quarks and gluons confine into hadrons, without deploying a phenomenological model. Here, we will adopt the two-stage model which adds a phenomenological hadronization stage to perturbative QCD calculations. Essentially, this model states that after the hard scattering of particles in the initial stage, a hadronization process occurs. This model allows for the inclusion of confinement effects and describes the multiplicity distribution for various processes, such as electron-positron annihilation.

Particle multiplicity is a central concept to understanding the physics of elementary particles and their interaction properties. Multiplicity parameters obtained from experiments allow for testing theoretical phenomenological models such as the 2-stage model.

This essay is developed in the scope that multiplicity is a central concept in modern high-energy physics thus, providing a pedagogical introduction to the subject.

2 QCD Jets

After a RHIC, complicated out-of-equilibrium processes occur in the collision environment's path to hadronization. After the collision, the quarks and gluons fragments are collimated into a narrow cone (a jet) of hadrons such as protons, κ,Π, and neutrons among many other possible hadrons created during such collision, hence, by identifying and measuring jets, allows one to reconstruct the kinematics of the elementary QCD interactions after a RHIC.

Despite the Physics of jets being an active area of research, we will briefly describe how a jet is generated omitting further details. For a complete reference regarding the Physics of jets, see.[1]

A traveling quark can radiate a gluon with probability:

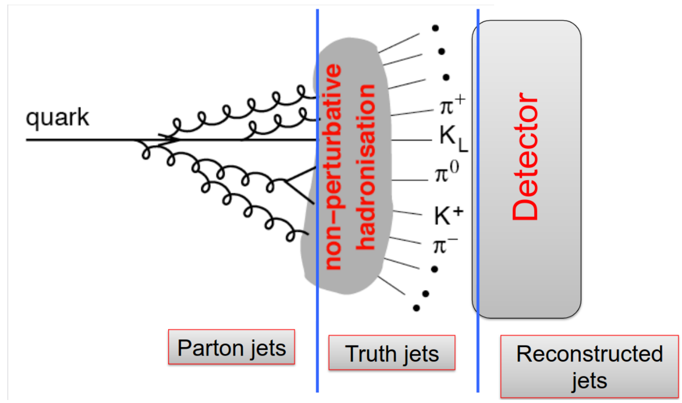

Figure 1: Idealization of a jet generation and its reconstruction. V.Cavasinni; U.Pisa and INFN

$$\int \alpha_s \frac{dE}{E}\frac{d\theta}{\theta} >> 1 \,, \tag{1}$$

in which α_s is the strong coupling of the theory and θ regards the angle of the gluon emission with respect to the quark movement direction. Note that this integral diverges if E or θ are small. The fact that it diverges for small energies is due to the prominent characteristic of QCD of asymptotic freedom. The probability of creating a set of jets is described by the jet production cross-section, an average of produced quarks, anti-quarks, and gluons weighted by the parton distribution function. The radiated gluon can also turn into a quark-antiquark pair, which will lead to the out-of-equilibrium hadronization step of the process. Figure 1 depicts an idealization of this process described here.

The probability of creating a set of jets is described by the jet production cross-section, an average of produced quarks, anti-quarks, and gluons weighted by the parton distribution function. In the most common jet pair production process, for instance, e^+e^- annihilation which is the one that we are interested in this study, this cross-section is given by:

$$\sigma_{ij\to k} = \sum_{ij}\int dx_1 dx_2 dt f_i^1(x_1,Q^2) f_j^2(x_2,Q^2)\frac{d\hat{\sigma}_{ij\to k}}{dt} \,. \tag{2}$$

In this expression, x_1 and x_2 stands for the longitudinal momentum fraction along the directions of the particle 1 or 2, while Q^2 is the longitudinal momentum transfer along these directions. The terms $f_i^a(x_a,Q^2)$ are the Parton distribution function of a particle species i at beam a.

Finally, let's recall the QCD evolution parameter in function of energy:

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$$Y(Q^2) = \frac{1}{2\pi b} ln\left[1 + b\alpha\ ln(\frac{Q^2}{\mu^2})\right] , \tag{3}$$

which describes the running of the coupling constant, α, with respect to a given energy scale μ^2. The term b regards to the QCD's β function. Clearly, one sees that α decreases as the energy involved in the process increases leading to the expected phenomenon of the weakening of the strong interaction at high energies, which in turn, leads to the deconfinement of the theory. Returning to expression 3, one sees that when Y=0, $Q^2 = \mu^2$, the value of the initial energy scale from which the parton begins to evolve. This quantity also allows us to extract the thickness of the jet which we will understand as the smallest scale to consider the parton as an entity. The jet thickness refers to the spatial or angular spread (the angular distribution of particles within the jet characterized by the jet radius parameter R is proportional to the cone size used to define the jet.) of a jet and is not a fixed quantity but rather a measure of how concentrated or diffuse the particles in a jet are, typically quantified by their angular spread around the jet axis. This quantity provides insights into the dynamics of Parton showering and hadronization, as well as the properties of the medium traversed by them, for instance, narrow jets are indicative of a highly collimated quark jet due to the high color charge leading to more radiation, while broad Jets may be due to gluon radiation or medium interactions, for instance, jets traversing a quark-gluon plasma tend to broaden and lose energy (jet quenching).

3 Multiplicity in a Heavy-Ion Collision

As relativistic high-energy physics experiments increased the accelerator's energy, new reaction channels regarding multiparticle processes were opened leading to a plethora of secondary particles related to the collision. Among the huge quantity of detectable observables in a RHIC, this study will focus on the multiplicity, the number of secondary charged and neutral particles generated in a given collision. This observable is a probe to investigate the extremely hot and dense system created right after the collision, during the out-of-equilibrium phase of overlap between the two incoming nuclei. Even without much information about the created particles, one can extract information about the collision from the total multiplicity of charged particles, for instance, the dependence on collision energy and centrality.[2] Furthermore, the study of events with the production of a large number of secondary particles is a tool to understand the hadronic interactions that occur in the hadronization stage. Multiparticle processes (MP) begin at energies of a few GeV per nucleon, in which MP starts to dominate over the elastic and quasi-elastic scattering processes favoring the already cited emergence of light mesons. At collision energies exceeding 100 GeV per nucleon, such as those achieved at RHIC and LHC (ALICE), the conditions for a nearly perfect fluid of deconfined quarks and gluons are reached and multiparticle production is dominated by soft processes, as well as hard scatterings leading to jets. At even higher energies, the ultra-high-energy collisions, for instance at LHC with energies ranging from 1 to 13 TeV/nucleon, multiplicity increases dramatically due to the copious production of soft gluons.

Is well known that at high energies, one can employ pQCD to describe the process of partonic fission, the cascade stage. After it, when partons lose part of their energy, they change into observed hadrons, a stage in which is not possible to employ pQCD. Thus, in order to describe the hadronization phenomenological models are used. It is usually difficult to determine the quark species on an event-by-event basis. For instance, the experimental results are averaged over the produced quark types.

Multiparticle processes have led to the discovery of jets, which we have discussed in the previous section. The most common processes that lead to partonic production are the e+e annihilation, deep inelastic scattering of e, μ or ν on nucleons, and hadron-hadron scattering, however, one of the most suitable processes for studying multiparticle processes is the e+e- annihilation study of MP. This process can be achieved by the following events:

$$e^+e^- \rightarrow (\gamma, Z_0) \rightarrow q\bar{q} \tag{4}$$

in which an electron and a positron (e^+e^-) annihilate, producing a quark-antiquark pair via the exchange of either a photon (γ) or a neutral Z boson (Z_0). The quark-antiquark pairs eventually form jets of hadrons, which are observable in detectors.

Statistical tools are fundamental to investigate not just the multiplicity of a given event but other observables as well. Regarding the multiplicity, an important statistical bound is events for which the deviation from the average multiplicity does not exceed two average values. Events with larger multiplicity are rare thus is hard to obtain relevant statistics from them.

4 Markovian Branching Process

A Markovian branching process models a population in which each element in a given *-nth* generation produces a random number of offspring for the next generation, *n+1*. Starting from a seed, it splits into *k* offsprings with a given probability which we will call P_k. These *k* offsprings constitute the 1st generation which, in turn, will independently split into a random number of offsprings. Let's define a Markov chain:

$$\{Z_n\} = Z_0, Z_1, ..., Z_k\ , \tag{5}$$

in which Z_n is a variable that describes the population's size at the -nth generation. In a given generation, the elements independently give rise to a certain number of offspring:

$$\xi_{n+1;1}, \xi_{n+1;2}..., \xi_{n+1;Z_n}\ , \tag{6}$$

in which the term ξ represents the number of members present in the $-nth$ generation where $\xi_{n,j}$ are offsprings of the $j-th$ member of the $n-1$ generation. Hence, the cumulative number of elements produced in the $n+1$ generation is:

$$Z_{n+1} = \xi_{n+1,1} + \xi_{n+1,2} + ... + \xi_{n+1,z_n}\ . \tag{7}$$

From these definitions, we can define the following generational sequence:

$$Z_0 = 1 \tag{8}$$
$$Z_1 = \xi_{1,1} \tag{9}$$
$$Z_2 = \xi_{2,1} + ... + \xi_{2,Z_1} \tag{10}$$
$$... \tag{11}$$
$$Z_n = \xi_{n,1} + ... + \xi_{n,Z_{n-1}}\ , \tag{12}$$

being an example of a Markovian branching process. Note that if in a given step $Z_k = 0$, then necessarily $Z_{k+1} = 0$.

With the ideas developed in these sections, we are able to introduce the processes of interest and how to investigate them within the scope of Markovian branching processes.

5 Processes of Interest

In this project, we are interested in 3 processes that contribute to the overall distribution of quark and gluon distributions inside QCD jets:

- A - Gluon fission: g→ g+g
- $\tilde{A}$ - Quark Bremssthralung: q→q+ g
- B - Quark pair creation: g→q+$\bar{q}$

The first process is due to the self-interaction nature of gluons contributing to the formation of gluon jets in high-energy particle collisions, such as those observed in proton-proton collisions at the LHC. The probability of this process increases at higher energy scales due to the running of the strong coupling constant. The quark bremsstrahlung implies a quark radiating a gluon as it accelerates or changes trajectory being a very similar process that occurs with photons in QED. This process plays a critical role in the formation of quark jets in high-energy collisions. The emitted gluon can then further interact or decay, contributing to the complexity of the final-state particle shower. The likelihood of gluon emission increases with energy but depends on angular distributions and phase space constraints. Finally, quark pair creation is a process in which a gluon transforms into a quark-antiquark pair. Quark-antiquark pairs play a role in forming hadrons, for instance, in the production of heavy quarks if the gluon energy exceeds the mass threshold, in other words, this process requires that the gluon has sufficient energy $E_g > 2m_q$ to create the pair.

From these processes, we may define the following quantities: $A\Delta Y$ as the probability that a gluon in the infinitesimal interval ΔY converts itself into two gluons; $\tilde{A}\Delta Y$ as the probability that a quark in the infinitesimal interval ΔY will radiate a gluon and $B\Delta Y$ as the probability that a quark-antiquark pair in the infinitesimal interval ΔY will be created from a gluon.

One must keep in mind that a quark in a jet will radiate gluons, which in turn will give origin to a quark-antiquark pair as discussed before. Note that the processes A,$\tilde{A}$ and B defined above are Y-independent and each quark or gluon element acts independently from the others with the same infinitesimal probability to convert themselves into m_q quarks and m_g gluons in the interval (Y, $\Delta Y + Y$). These probabilities are:

$$\delta_{1,m_g}\delta_{0,m_q} + a^g_{m_g,m_q}\Delta Y + \mathcal{O}(\Delta Y) \quad (13)$$

$$\delta_{0,m_g}\delta_{1,m_q} + a^q_{m_g,m_q}\Delta Y + \mathcal{O}(\Delta Y) \quad (14)$$

Some words are necessary to explain the meaning of these expressions: the terms at the left side represent the initial situation in which there are two probabilities associated with one gluon and no quarks in the initial state, respectively, thus the δ is the initial state probabilities. Now, we must discuss the meaning of the right-side terms. For this, start recalling that we are considering only the processes A, $\tilde{A}$ and B, so we can identify $A = a^g_{20}$, $B = a^g_{02}$ and $\tilde{A} = a^q_{11}$. Furthermore, $\delta_{1,mg} = 1$ if $m_g = 1$, one gluon in the final state and zero otherwise, and $\delta_{0,mq} = 1$ if $m_q = 0$, zero quarks in the final state and zero otherwise. Hence, by employing the conservation of probability we have for the gluon parts:

$$1 + a^g_{1,0}\Delta Y + a^g_{2,0}\Delta Y + a^g_{0,2}\Delta Y \implies a_{1,0} + a_{2,0} + a_{0,2} = 0\ , \quad (15)$$

from this, $a_{1,0} < 0$, for instance, $a_{1,0} = -a_{2,0} - a_{0,2}$. Now, for the quark part:

$$1 + a^q_{0,1}\Delta Y + a^q_{1,1}\Delta Y \implies a_{1,0} + a_{0,1} = 0\ , \quad (16)$$

from this, $a_{0,1} < 0$, for instance, $a_{0,1} = -a_{1,1}$. In fact, $1 - a^g_{20} - a^g_{02} = 1 + a^g_{10}$ and $1 - a^q_{11} = 1 + a^q_{11}$ are probabilities for the quark or gluon which gave origin on the corresponding jet to continue undisturbed on their way without converting into anything.

Now we can obtain generating functions that represent the probability distributions of the quark and gluon emissions multiplicity.

6 Generating Functions

The generation functions allow us to study the evolution of these multiplicities with respect to a continuous variable, which in this study we will consider the already defined Y. From the definition of generating functions:

$$G_{q,g}(Y, z) = \sum_n P_{q,g}(Y, n) z^n \quad (17)$$

The reference[3] introduces as auxiliary functions:

$$\begin{aligned} w^g(u_g, u_q) &= \sum_{m_g,m_q}^{\infty} a^g_{mg} a^q_{mq} u_g^{mg} u_q^{mq} \\ &= (-A - B)u_g + Au_g^2 + Bu_q^2 \\ w^q(u_g, u_q) &= -\tilde{A}u_q + \tilde{A}u_q u_g\ , \end{aligned} \quad (18)$$

Now, defining $P_{mg,mq;ng,nq}(Y)$ as the probability that m_g and m_q will be transformed into n_g and n_q gluons and quarks, respectively, we can write the generating functions for the gluons and quarks as:

$$G(ug, uq, Y) = \sum_{ng,nq=0}^{\infty} P_{1,0;ng;nq}(Y) u_g^{ng} u_q^{nq} \quad (19)$$

$$Q(ug, uq, Y) = \sum_{ng,nq=0}^{\infty} P_{0,1;ng;nq}(Y) u_g^{ng} u_q^{nq} \quad (20)$$

In which the term $P_{1,0;ng;nq}(Y)$ represents the probability of a single gluon and zero quarks at Y=0 and $P_{0,1;ng;nq}(Y)$ represents the probability of a single quark and zero gluons at Y=0. Since the partons are independent, the probability is:

$$\sum_{ng,nq=0}^{\infty} P_{mg;mq;nq,ng}(Y) u_g^{ng} u_q^{nq} = \left[G(u_g, u_q; Y)\right]^{mg} \left[Q(u_g, u_q; Y)\right]^{mq} \quad (21)$$

Due to the partonic independence, the process is homogenous in Y, so the transition probabilities obey the Chapman-Kolmogorov equations, which relate the joint probability distributions of different coordinate sets on a stochastic process. Suppose that $P_{i1,...,in-1}(f_1,...,f_n)$ is the joint probability distribution function of the possible values $f_1,...,f_n$, so the Chapman-Kolmogorov equation is given by:

$$P_{i1,...,in-1}(f_1,...,f_n) = \int_{-\infty}^{\infty} P_{i1,...,in-1}(f_{1i},...,f_n)df_n \quad (22)$$

From,[3] the Chapman-Kolmogorov equation is:

$$P_{mg,mq;ng,nq}(Y+Y') = \sum_{lg,lq=0}^{\infty} P_{mg,mq;lg,lq}(Y)P_{lg,lq;nq,ng}(Y') \quad (23)$$

Following,[3] it is not hard to obtain the forward and backward Kolmogorov equations for the generating functions of the transition probabilities $P_{mg,mq;ng,ng}$ which allow us to completely solve our problem. Following[3] and recalling that only the processes of quark Bremsthralung, gluon fission, and quark pair creation processes are allowed, we obtain the following set of coupled equations:

$$\frac{\partial G}{\partial Y} = -AG + AG^2 - BG + BQ^2 \quad (24)$$
$$\frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG \quad (25)$$

Recall that A, B and $\tilde{A}$ refer to the allowed processes described above. Imposing the initial conditions:

$$G(u_g,u_q,0) = u_g \quad (26)$$
$$Q(u_g,u_q,0) = u_q \ , \quad (27)$$

we completely solve our problem. However, there's an alternative approach to solving our problem in terms of the transition probability functions, for instance, the normalized exclusive cross sections for producing n_g gluons and n_q quarks without referring to the generating functions. By assuming initial conditions such as the presence or not of one gluon and no quark or of one quark at Y = 0, we can build the probability for a gluon or a quark to produce, in the interval (Y + ΔY), n_g and n_q quarks via the allowed processes here mentioned by imposing conservation of probability. According to,[3] this probability for a gluon jet is:

$$P_{1,0;n_g,n_q}(Y+\Delta Y) = \left[1 - \tilde{A}n_q\Delta Y - An_g\Delta Y - Bn_g\Delta Y\right] * P_{1,0;n_g,n_q}(Y) + \tilde{A}n_q\Delta Y P_{1,0;ng-1,nq}(Y) + B(n_g-1)\Delta Y P_{1,0;ng+1,nq-2}(Y) + \mathcal{O}\Delta Y \quad (28)$$

Dividing it by ΔY and letting $\Delta Y \to 0$ we get a system of differential equations equipped with the initial conditions given above at Y=0. Hence, according to,[3] normalized exclusive cross sections for gluons and quarks produced in a gluon jet can be determined in terms of the allowed processes, in other words, in terms of the average partonic multiplicity. The procedure leads us to the set of equations:

$$\frac{\partial G}{\partial Y} = AG^2 - AG - BG \quad (29)$$
$$\frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG \quad (30)$$

Note the resemblance with the set of equations previously obtained establishing an equivalence between the two approaches.

Thus, in the next section, we will explore some an approximate solution that can be obtained for the specific case of a process without quark pair creation from gluons.

7 Solutions for Gluon Fission and Quark Bremssthralung

In this work, we will investigate just the solution for the already mentioned processes of gluon fission and quark Bremsstrahlung thus, we will not investigate the possibility of gluons splitting into quark-antiquark pairs, thus we will not consider flavors in the theory. Recall that:

$$\frac{dP_{1,0;ng,0}(Y)}{dY} = -A_{ng}P_{1,0;ng,0}(Y) + A(ng-1)P_{1,0;ng-1,0}(Y) \quad (31)$$

$$\frac{dP_{0,1;ng,1}(Y)}{dY} = -\tilde{A}P_{0,1;ng,1}(Y) - AngP_{0,1;ng,1}(Y) + \quad (32)$$
$$\tilde{A}P_{0,1;ng-1,1}(Y) + A(ng-1)P_{0,1;ng-1,1}(Y) \quad (33)$$

With the initial conditions:

- $P_{1,0;1,0}(0) = 1$; $P_{1,0;ng,0}(0) = 0$; $\forall ng > 1$
- $P_{0,1;0,1}(0) = 1$; $P_{0,1;ng,1}(0) = 0$; $\forall ng \geq 1$

Solving recursively this set of equations with the given set of initial conditions for different numbers of n_g, we obtain that:

$$P_{1,0;1,0}(Y) = e^{-AY} \quad (34)$$
$$P_{1,0;ng,0}(Y) = e^{-AY}(1-e^{-AY})^{ng-1} \quad (35)$$

And the mean value:

$$\langle n_g \rangle = \left.\frac{\partial G}{\partial u_g}\right|_{ug=1} = e^{AY} \quad (36)$$

Using the results derived we obtain that:

$$G(u_g, Y) = \frac{u_g}{\langle n_g\rangle[1 - ng(1 - \frac{1}{\langle ng\rangle})]} \quad (37)$$

Finally, the variance:

$$D^2 = \langle n_g^2\rangle - \langle n_g\rangle^2 = e^{AY}(e^{AY}-1) \quad (38)$$

Analogously, the quark jet is:

$$P_{0,1;0,1}(Y) = e^{-\tilde{A}Y} \quad (39)$$
$$P_{0,1;ng,1} = \frac{\mu(\mu+1)...(\mu+ng-1)}{ng!}e^{-\tilde{A}Y}(1-e^{-AY})^{ng} \quad (40)$$

which resembles a Polya-Eggenberger distribution and $\mu = \tilde{A}/A$ and, as before, the average gluon multiplicity in the quark jet will be:

$$\langle n_g \rangle = \mu(e^{AY} - 1) \tag{41}$$

And the normalized exclusive cross-section for producing n_g gluons is given by:

$$\frac{\sigma_{ng}}{\sigma_{tot}} = \frac{\mu(\mu+1)...(\mu+ng-1)}{ng!} \left[\frac{\langle ng \rangle}{\langle ng \rangle + \mu}\right]^{ng} \left[\frac{\mu}{\langle ng \rangle + \mu}\right]^{\mu} \tag{42}$$

Thus, the generating function is:

$$Q = \sum_{ug=0}^{\infty} u_g^{ng} u_q P_{0,1;ng,1}(Y) = u_q \left[\frac{e^{-AY}}{1 - u_g(1 - e^{-AY})}\right]^{\mu} \tag{43}$$

As before the variance:

$$D^2 = \mu e^{AY}(e^{AY} - 1) \tag{44}$$

8 Hadronization

Our goal is to adopt the already discussed two-stage method to describe the phenomenon of interest, the annihilation of electron-positron pairs at high energies. Recall that this process consists of two main stages: the cascade stage and the hadronization stage. Firstly, we write the generating function as:

$$P_n(s) = \sum_{ng} P_{ng} P_{had}(ng, s) \tag{45}$$

In which the first term on the right refers to the partons and the other term refers to the hadrons produced from m partons. The stage of partonic fission is described by a Polya-Eggenberger distribution which we reproduce here for the sake of readability:

$$\frac{\sigma_{ng}}{\sigma_{tot}} = \frac{\mu(\mu+1)...(\mu+ng-1)}{ng!} \left[\frac{\langle ng \rangle}{\langle ng \rangle + \mu}\right]^{ng} \left[\frac{\mu}{\langle ng \rangle + \mu}\right]^{\mu} \tag{46}$$

In the next section, we will conclude the discussion regarding the average multiplicity of the event of interest - e^+e annihilation - by using experimental results.

9 Experimental Results

The data on multiplicity distribution were taken from the TASSO detector at the PETRA accelerator, obtained in 1989 at center-of-mass energies 14GeV, 22GeV, 34.8Gev and 46.8GeV.[4] In the following plots, we reproduce the obtained data with the respective statistical errors from the correction procedure.

First of all, it's important to see that the data is presented in the shape of the proposed distribution thus, a further study is to compare the data with the obtained discussed distribution, the negative binomial distribution (NBD). This was already done in.[4] One sees that the statistical distribution describes the data very well for low energies. A deep discussion regarding the phase space region of validity of the fit is also done in.[4] In general, there is good agreement between the data and the shape of the proposed distribution.

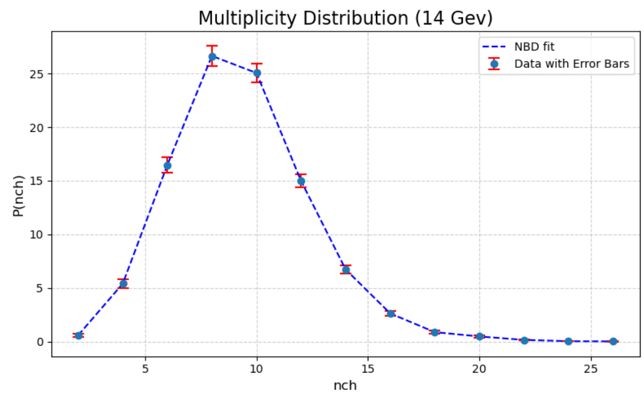

Figure 2: Multiplicity Distribution at 14GeV for e^+e annihilation event. Data taken from[4]

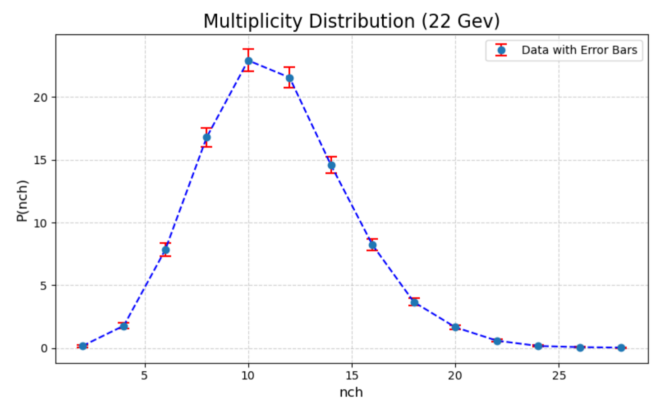

Figure 3: Multiplicity Distribution at 22GeV for e^+e annihilation event. Data taken from[4]

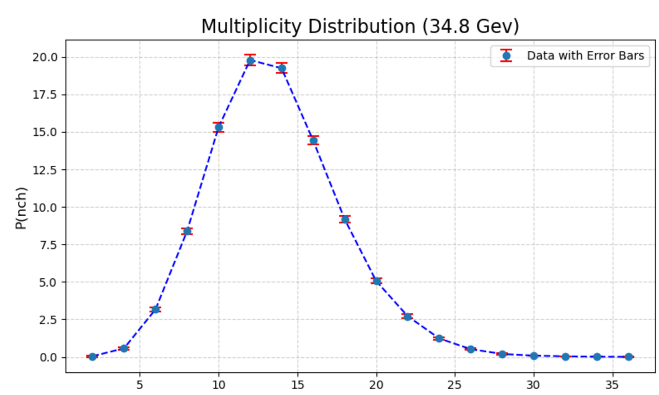

Figure 4: Multiplicity Distribution at 34.8GeV for e^+e annihilation event. Data taken from[4]

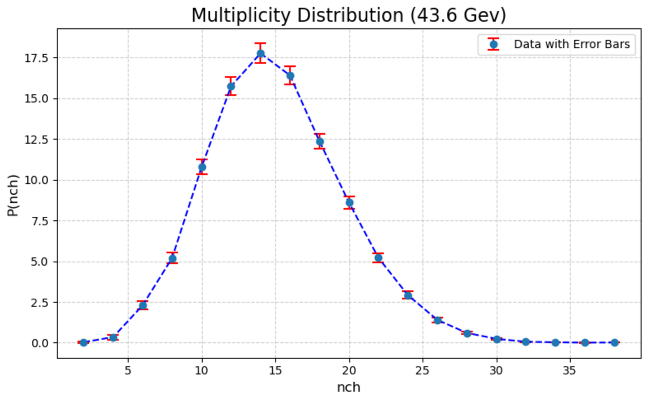

Figure 5: Multiplicity Distribution at 43.6GeV for e^+e annihilation event. Data taken from[4]

10 Conclusion

As we have discussed, one of the main parameters that one may obtain from high-energy experiments is the multiplicity, the number of secondary particles generated in such event. A relevant tool to investigate such phenomenology is perturbative QCD, which enables the calculation of hard processes in particle interactions. However, in order to describe it encounters challenges in describing the hadronization stage, a highly out-of-equilibrium stage of a high-energy event, in which quarks and gluons combine themselves to form hadrons. One of the tools developed to address this issue is to employ a two-stage model involving the addition of a phenomenological hadronization stage to the calculations. This model facilitates the computation of multiplicity distribution for processes such as electron-positron annihilation, the one that we investigated in this report. Using this model,

In order to check the theoretical results, we reproduced the experimental data from TASSO collaboration[4] to fit the data obtaining the Multiplicity Distribution for 14, 22, 34.8, and 43.6 GeV. We see that the data reproduces the shape of the discussed NB distribution, suggesting a further statistical investigation of the data employing this distribution. Furthermore, we

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