JOINT INSTITUTE FOR NUCLEAR RESEARCH

Veksler and Baldin laboratory of High Energy Physics

FINAL REPORT ON THE INTEREST PROGRAMME

Active role of gluons in hadron interactions (part II)

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Participation period:
05 November - 14 December,
Wave 11

Dubna, 2024

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1. Introduction

One of the observables in the high energy experiment is multiplicity - the number of secondary particles. The most popular one is the multiplicity of charged particles. There are more and more accelerators and projects with higher and higher energy. With energy increasing new channels of decays were discovered, new particles were born. The quantum chromodynamics (QCD) was created to study strong interaction and to provide deep understanding of matter and energy. QCD allows us to calculate hard processes of particle interactions at the amplitude level. However, dificulties arise when describing the hadronization stage, when quarks and gluons confine into hadrons. To account for this phase, a two-stage model is proposed, which adds a phenomenological hadronization stage to pQCD calculations.

Multiparticle production (MP) constitutes a significant domain within high-energy physics, and the study of multiparticle production serves as a vital testing ground for QCD. The annihilation process of electron-positron pairs (Figure 1) stands out as one of the most effective means to investigate MP phenomenon. When an electron collides with a positron, they can annihilate into either a virtual photon or a Z^0 boson. Both the virtual photon and the Z^0 boson subsequently decay into pairs of fermions and anti-fermions, specifically quarks and antiquarks.

$$e^+e^- \to \gamma\gamma/Z^0 \to q\bar{q}$$

The first stage of fission partons at high energy is called a stage of cascade. The electron and positron annihilate into a virtual photon or Z^0 boson. These virtual particles then decay into pairs of fermions and antifermions, including quarks and antiquarks.

The second stage – a stage of hadronization. Hadronization describes the conversion of quarks and gluons into hadrons. It is important to note that unlike the first stage, the hadronization stage cannot be described by perturbative quantum chromodynamics (QCD) due to the low energies of the particles involved.

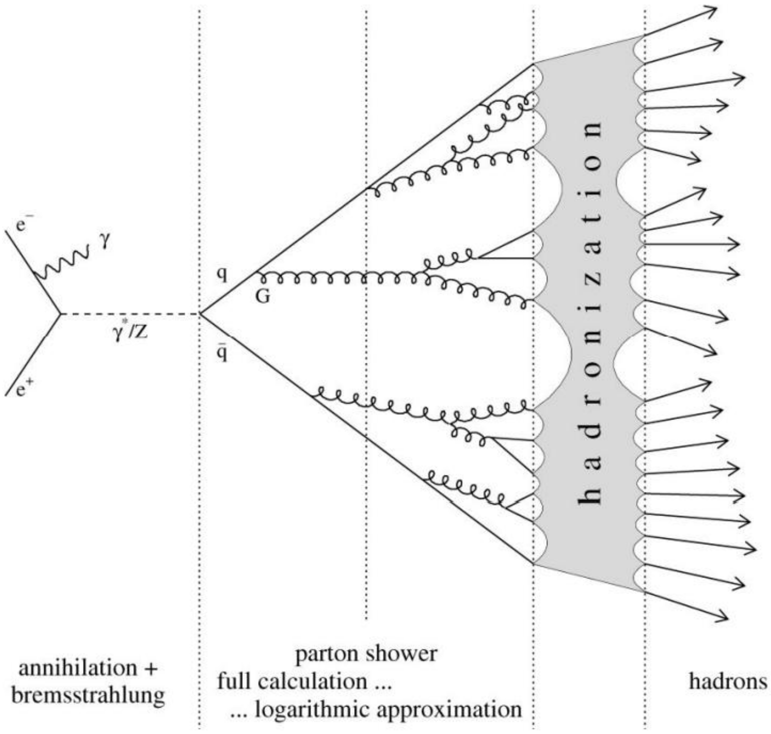

Figure 1: Diagram of e^+e^- annihilation

As a result, the two-stage process of electron-positron pair annihilation illustrates a sequential progression of events, beginning with the high-energy splitting of particles in the first stage and culminating in the production of observable hadrons in the second stage. This comprehensive journey provides valuable insights into multiparticle production mechanisms. Each stage is crucial for understanding the processes involved in particle multiplicity generation and remains a subject of ongoing research in elementary particle physics.

2. Cascade stage

To study multiparticle production we used approach of A. Giovannini [1]. The main idea is to describe quark and gluon jets and their development through subnuclear matter as Markov branching processes.

The idea of applying branching Markov processes to Quantum Chromodynamics (QCD) in Giovannini's work is that the evolution of jets formed in high-energy particle collisions can be described as a sequence of random branchings that follow certain probability laws.

It was proposed to interpret the natural QCD evolution parameter:

$$Y = \frac{1}{2\pi b}\log[1+\alpha\beta\log\frac{Q^2}{\mu^2}] \tag{2.1}$$

where $2\pi b = \frac{1}{6}(11N_c - 2N_f)$ for a theory with N_c colors and N_f flavors, as the thickness value of the QCD jets).

There are three main elementary processes that contribute to the overall gluon or quark distribution inside QCD jets with different weights:

- **gluon fission:** $g \to g + g$, with $A\Delta Y$ denoting the probability that a gluon in the infinitesimal interval ΔY will convert into two gluons. (*)
- **quark bremsstrahlung:** $q \to q + g$, with $\tilde{A}\Delta Y$ denoting the probability that a quark in the infinitesimal interval ΔY will radiate a gluon, the quark continuing on its way. (**)
- **quark pair creation:** $g \to q + \bar{q}$, with $B\Delta Y$ denoting the probability that a quark-antiquark pair will be created from a gluon in the infinitesimal interval ΔY. (***)

In addition, A, $\tilde{A}$, B are assumed to be Y-independent constants and each individual parton (quark or gluon) acts independently from the others, always with the same infinitesimal probability.

The probability for a gluon or quark to convert into m_q quarks and m_g gluons in the (Y, Y+ΔY) can be given by sum of probabilities

(gluon) $$\delta_{1,m_g}\delta_{0,m_q} + a^{(g)}_{m_g,m_q}\Delta Y + l(\Delta Y) \quad (2.2)$$

(quark) $$\delta_{0,m_g}\delta_{1,m_q} + a^{(q)}_{m_g,m_q}\Delta Y + l(\Delta Y) \quad (2.3)$$

Due to only 3 processes are being allowed in the same interval ΔY, we get for gluon (2.4) and in case of quark (2.5)

$$1 + a^{(g)}_{1,0}\Delta Y + a^{(g)}_{2,0}\Delta Y + a^{(g)}_{0,2}\Delta Y + l(\Delta Y) \quad (2.4)$$

$$1 + a^{(q)}_{0,1}\Delta Y + a^{(q)}_{1,1}\Delta Y + l(\Delta Y) \quad (2.5)$$

Notice that $a^{(g)}_{1,0} + a^{(g)}_{2,0} + a^{(g)}_{0,2} = 0$ and $a^{(q)}_{0,1} + a^{(q)}_{1,1} = 0$ because of probability conservation. Let's identify $a^{(g)}_{2,0}$ as A, $a^{(g)}_{0,2}$ as Ã, and $a^{(q)}_{1,1}$ as B.

After this, the infinitesimal function for gluon (2.6) and quark (2.7) jets are introduced

$$w^{(g)}(u_g,u_q) = \sum_{m_g,m_q=0}^{\infty} a^{(g)}_{m_g,m_q} u_g^{m_g} u_q^{m_q} = a^{(g)}_{2,0}u_g^2 + a^{(g)}_{0,2}u_q^2 + a^{(g)}_{1,0}u_g$$

$$w^{(g)}(u_g,u_q) = Au_g^2 + Bu_q^2 - (A+B)u_g \quad (2.6)$$

$$w^{(q)}(u_g,u_q) = \sum_{m_g,m_q=0}^{\infty} a^{(q)}_{m_g,m_q} u_g^{m_g} u_q^{m_q} = a^{(q)}_{1,0}u_q + a^{(q)}_{1,1}u_g u_q$$

$$w^{(q)}(u_g,u_q) = \tilde{A}(u_g u_q - u_q) \quad (2.7)$$

Giovannini defines next the probability that m_g gluons and m_q quarks will be transformed into n_g gluons and n_q quarks over a jet of thickness Y and calls it $Pm_gm_qn_gn_q$(Y).

Probability generating function for a gluon jet (2.8) and a quark jet (2.9) will be

$$G(u_g, u_q; Y) = \sum_{n_g, n_q = 0}^{\infty} P_{1,0;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} \tag{2.8}$$

$$Q(u_g, u_q; Y) = \sum_{n_g, n_q = 0}^{\infty} P_{0,1;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} \tag{2.9}$$

Action of different partons are independent: from a probabilistic point of view the total m_g gluons and m_q quarks populations are evolving as (m_g + m_q) independent parton populations, each with one initial quark or gluon. This fact summarizes the branching Markov chain nature of the process. It can be shown through straightforward calculations that

$$\sum_{n_g, n_q = 0}^{\infty} P_{m_g,m_q;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} = [G(u_g, u_q; Y)]^{m_g} [Q(u_g, u_q; Y)]^{m_q} \tag{2.10}$$

Moreover, since the process is homogenous in Y the transition probabilities obey Chapman-Kolmogorov equations in general case

$$P_{m_g,m_q;n_g,n_q}(Y + Y') = \sum_{l_g, l_q = 0}^{\infty} P_{m_g,m_q;l_g,l_q}(Y) P_{l_g,l_q;n_g,n_q}(Y') \tag{2.11}$$

And for the gluon jet

$$P_{1,0;n_g,n_q}(Y + Y') = \sum_{l_g, l_q = 0}^{\infty} P_{1,0;l_g,l_q}(Y) P_{l_g,l_q;n_g,n_q}(Y') \tag{2.12}$$

And for the quark jet

$$P_{0,1;n_g,n_q}(Y + Y') = \sum_{l_g, l_q = 0}^{\infty} P_{0,1;l_g,l_q}(Y) P_{l_g,l_q;n_g,n_q}(Y') \tag{2.13}$$

Using these additional equations (2.10), (2.12) and (2.13), we can obtain a probability generating function for gluons

$$G(u_g,u_q;Y+Y') = \sum_{l_g,l_q=0}^{\infty} P_{1,0;l_g,l_q}(Y+Y')u_g^{l_g}u_q^{l_q}$$

$$= \sum_{l_g,l_q=0}^{\infty}\left\{\sum_{n_g,n_q=0}^{\infty} P_{1,0;n_g,n_q}(Y)P_{n_g,n_q;l_g,l_q}(Y')\right\}u_g^{l_g}u_q^{l_q}$$

$$= \sum_{n_g,n_q=0}^{\infty} P_{1,0;n_g,n_q}(Y)\left\{\sum_{l_g,l_q=0}^{\infty} P_{n_g,n_q;l_g,l_q}(Y')u_g^{l_g}u_q^{l_q}\right\}$$

$$= \sum_{n_g,n_q=0}^{\infty} P_{1,0;n_g,n_q}(Y)\left[G(u_g,u_q;Y')\right]^{n_g}\left[Q(u_g,u_q;Y')\right]^{n_q}$$

$$G(u_g,u_q;Y+Y') = G\left[G(u_g,u_q;Y'),Q(u_g,u_q;Y');Y\right] \qquad (2.14)$$

And a probability generating function for quarks can be obtained analogically

$$Q(u_g,u_q;Y+Y') = Q\left[G(u_g,u_q;Y'),Q(u_g,u_q;Y');Y\right] \qquad (2.15)$$

We can rewrite it using $P_{1,0;1,0}(\Delta Y) = 1$, $P_{0,1;0,1}(\Delta Y) = 1$

$$G(u_g,u_q;\Delta Y) = u_g + w^{(g)}(u_g,u_q)\Delta Y + o(\Delta Y) \qquad (2.16)$$

$$Q(u_g,u_q;\Delta Y) = u_q + w^{(q)}(u_g,u_q)\Delta Y + o(\Delta Y) \qquad (2.17)$$

Inserting (2.16), (2.17) into (2.14), (2.15) and replacing Y' with ΔY, then dividing both sides by ΔY and taking the limit as $\Delta Y \to 0$, we can obtain:

$$\frac{\partial G(u_g,u_q;Y)}{\partial Y} = \frac{\partial G}{\partial u_g}w^{(g)}(u_g,u_q) + \frac{\partial G}{\partial u_q}w^{(q)}(u_g,u_q) \qquad (2.18)$$

And for quarks analogically

$$\frac{\partial Q(u_g,u_q;Y)}{\partial Y} = \frac{\partial Q}{\partial u_g}w^{(g)}(u_g,u_q) + \frac{\partial Q}{\partial u_q}w^{(q)}(u_g,u_q) \qquad (2.19)$$

(2.18) and (2.19) are the forward Kolmogorov equations for the generating functions of the transition probabilities $P m_g m_q n_g n_q(Y)$. But for solving our problem it's necessary to get corresponding backward Kolmogorov equations, which immediately follows from inserting (2.16), (2.17) into (2.14), (2.15) and letting $Y \to \Delta Y$. And then after dividing both sides by $\Delta Y \to 0$ we obtain

$$\frac{\partial G}{\partial Y} = w^{(g)}\left[G\left(u_g, u_q; Y\right), Q\left(u_g, u_q; Y\right)\right] \quad (2.20)$$

$$\frac{\partial Q}{\partial Y} = w^{(q)}\left[G\left(u_g, u_q; Y\right), Q\left(u_g, u_q; Y\right)\right] \quad (2.21)$$

For both cases of Kolmogorov equations there are initial conditions that are given by

$$G|_{Y=0} = u_g \quad (2.22)$$

$$Q|_{Y=0} = u_q \quad (2.23)$$

After this, using (2.6) and (2.7) our equations become

$$\frac{\partial G}{\partial Y} = -AG - BG + AG^2 + BQ^2 \quad (2.24)$$

$$\frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG \quad (2.25)$$

The general theory presented here can be applied to the hypothetical situation where we have to deal with processes more complex than those indicated in (*)-(***) (e.g., g → g + g + g or q → q + g + g ...).

We need only to identify

$$P_{1,0;n_g,n_q}(Y) \equiv \frac{\sigma(g \to n_g + n_q)}{\sigma_{total}}$$

$$P_{0,1;n_g,n_q}(Y) \equiv \frac{\sigma(q \to n_g + n_q)}{\sigma_{total}}$$

Given certain initial conditions (i.e., the presence of one gluon and no quark or of one quark and no gluon at Y = 0) we can ask ourselves what is the probability for a gluon or a quark to produce in the interval (Y+ΔY) n_g gluons and n_q quarks through processes (*)-(***) under the requirement of probability conservation.

Considering that only 3 processes are allowed, and at the same time nothing can happen, it follows for a gluon jet

$$\begin{aligned} P_{1,0;n_g,n_q}(Y+\Delta Y) = \tilde{A}\Delta Y P_{1,0;n_g-1,n_q}(Y)\cdot n_q + A\Delta Y P_{1,0;n_g-1,n_q}(Y)\cdot(n_g-1) + \\ +B\Delta Y P_{1,0;n_g+1,n_q-2}(Y)\cdot(n_g+1) + \\ +(1-\tilde{A}\Delta Y n_q - A\Delta Y n_g - B\Delta Y n_g)P_{1,0;n_g,n_q}(Y) + o(Y) \end{aligned} \tag{2.26}$$

And for a quark jet

$$\begin{aligned} P_{0,1;n_g,n_q}(Y+\Delta Y) = \tilde{A}\Delta Y P_{0,1;n_g-1,n_q}(Y)\cdot n_q + A\Delta Y P_{0,1;n_g-1,n_q}(Y)\cdot(n_g-1) + \\ +B\Delta Y P_{0,1;n_g+1,n_q-2}(Y)\cdot(n_g+1) + \\ +(1-\tilde{A}\Delta Y n_q - A\Delta Y n_g - B\Delta Y n_g)P_{0,1;n_g,n_q}(Y) + o(Y) \end{aligned} \tag{2.27}$$

Dividing by ΔY and letting ΔY → 0 we obtain the system of differential equations.

$$\begin{aligned} \frac{dP_{1,0;n_g,n_q}(Y)}{dY} = (-\tilde{A}n_q - An_g - Bn_g)P_{0,1;n_g,n_q}(Y) + \\ + \tilde{A}P_{0,1;n_g-1,n_q}(Y)\cdot n_q + AP_{0,1;n_g-1,n_q}(Y)\cdot(n_g-1) + \\ + BP_{0,1;n_g+1,n_q-2}(Y)\cdot(n_g+1) \end{aligned} \tag{2.28}$$

We are only interested in the gluon exclusive cross sections both in a gluon- or a quark-jet, i.e., in $P_{1,0;\,n_g,0}$ (Y) and $P_{0,1;\,n_g,1}$ (Y), and for this reason we obtain

$$\frac{dP_{1,0;n_g,0}(Y)}{dY} = -(B+A)n_gP_{1,0;n_g,0}(Y) + A(n_g-1)P_{1,0;n_g-1,0}(Y) \tag{2.29}$$

$$\begin{aligned} \frac{dP_{0,1;n_g,1}(Y)}{dy} = -\tilde{A}P_{0,1;n_g,1}(Y) - (B+A)n_gP_{0,1;n_g,1}(Y) + \\ +\tilde{A}P_{0,1;n_g-1,1}(Y) + A(n_g-1)P_{0,1;n_g-1,1}(Y) \end{aligned} \tag{2.30}$$

After this, we want to explicit solutions in particular cases. Thus, we consider that $B = 0$, $A \neq \tilde{A} \neq 0$. It means we forbid gluons from splitting into quark-antiquark pair (or absence of flavours in theory). Equations for the cross sections for n_g gluons in the gluon- and quark- jet will be

$$\frac{dP_{1,0;n_g,0}(Y)}{dY} = -An_g P_{1,0;n_g,0}(Y) + A(n_g - 1)P_{1,0;n_g-1,0}(Y) \tag{2.31}$$

$$\begin{aligned} \frac{dP_{0,1;n_g,1}(Y)}{dy} = -\tilde{A}P_{0,1;n_g,1}(Y) - An_g P_{0,1;n_g,1}(Y) + \\ +\tilde{A}P_{0,1;n_g-1,1}(Y) + A(n_g - 1)P_{0,1;n_g-1,1}(Y) \end{aligned} \tag{2.32}$$

with initial conditions

$$P_{1,0;n_g,0}(0) = \delta_{1n_g} \tag{2.33}$$

$$P_{0,1;n_g,1}(0) = \delta_{0n_g} \tag{2.34}$$

1) For the gluon jet we get from (2.1) and (2.33)

$$P_{1,0;1,0}(Y) = e^{-AY} \tag{2.35}$$

$$P_{1,0;n_g,0}(Y) = e^{-AY}(1 - e^{-AY})^{n_g-1} \tag{2.36}$$

The corresponding generating function is

$$G = \sum_{n_g=1}^{\infty} u_g^{n_g} P_{1,0;n_g,0}(Y) = \frac{u_g e^{-AY}}{1 - u_g(1 - e^{-AY})} \tag{2.37}$$

Additionally, since

$$\langle n_g \rangle = \frac{\partial G}{\partial u_g} \Big|_{u_g=1} = e^{AY} \tag{2.38}$$

Therefore, we have a normalized cross section

$$\frac{\sigma_{n_g,0}^{(g,0)}}{\sigma_{total}} \equiv P_{1,0;n_g,0}(Y) = \frac{1}{\langle n_g \rangle}\left(1 - \frac{1}{\langle n_g \rangle}\right)^{n_g-1} \tag{2.39}$$

2) For the quark jet we get ($\mu = \frac{\tilde{A}}{A}$)

$$P_{0,1;0,1}(Y) = e^{-\tilde{A}Y} \tag{2.40}$$

$$P_{0,1;n_g,1}(Y) = \frac{\mu(\mu+1)\dots(\mu+n_g-1)}{n_g!}e^{-\tilde{A}Y}(1-e^{-AY})^{n_g} \tag{2.41}$$

The average gluon multiplicity in the quark jet will be

$$\langle n_g \rangle = \frac{\partial Q}{\partial u_g}\Big|_{u_g=1} = \mu(e^{AY}-1) \tag{2.42}$$

And the normalized exclusive cross section for producing n_g gluons

$$\begin{aligned}\frac{\sigma_{n_g,0}^{(0,q)}}{\sigma_{total}} &\equiv P_{0,1;n_g,1}(Y) = \\ &= \frac{\mu(\mu+1)\dots(\mu+n_g-1)}{n_g!}\left(\frac{\langle n_g \rangle}{\langle n_g \rangle+\mu}\right)^{n_g}\left(\frac{\mu}{\langle n_g \rangle+\mu}\right)^{\mu}\end{aligned} \tag{2.43}$$

To sum up, (2.43) is a Polya-Eggenberger distribution, which in the limit $N_c \to \infty$ assumes half-integer μ values, whereas (2.39) is a Furry-Yule distribution, corresponding to a Polya-Eggenberger distribution with $\mu = 1$.

3. Hadronization stage

By combining the two stages, we obtain the generating function as

$$P_n(s) = \sum_m P_m^P P_n^H(m,s) \tag{3.1}$$

Where P_m^P is multiple distribution (MD) for partons (2.43) (from this moment m is equal to n_g), P_n^H - MD for hadrons produced from m partons on the stage of hadronization.

The stage of hard fission of partons is characterized by a negative binomial distribution (NBD) for quark jet.

$$P_m^P(s) = \frac{k_p(k_p+1)\ldots(k_p+m-1)}{m!}\left[\frac{\bar{m}}{\bar{m}+k_p}\right]^m \left[\frac{k_p}{\bar{m}+k_p}\right]^{k_p} \tag{3.2}$$

Where $k_p = \frac{\tilde{A}}{A}$ and $\bar{m} = \sum_m m\, P_m^P$.

P_m^P and generating function for MD Q_m^P (s, z) are

$$P_m^P = \frac{1}{m!}\frac{\partial^m}{\partial z^m} Q^P(s,z)|_{z=0} \tag{3.3}$$

$$Q_m^P(s,z) = \left[1 + \frac{\bar{m}}{k_p}(1-z)\right]^{-k_p} \tag{3.4}$$

P_n^H and generating function for MD Q_p^H (s, z) are

$$P_n^H = C_{N_p}^n \left[\frac{\bar{n}_p^h}{N_p}\right]^n \left[1 - \frac{\bar{n}_p^h}{N_p}\right]^{N_p-n} \tag{3.5}$$

$$Q_p^H = \left[1 + \frac{\bar{n}_p^h}{N_p}(z-1)\right]^{N_p} \tag{3.6}$$

$\bar{n}_p^h$ is an average multiplicity formed from parton on the stage of hadronization.

N_p is maximum secondary of hadrons from parton on the stage of hadronization.

MD of hadrons in e^+e annihilation are determined by convolution of two stages: cascade and hadronization (3.1).

$$P_n(s) = \sum_m P_m^P \frac{\partial^n}{\partial z^n}(Q^H)^{2+m}|_{z=0} \tag{3.7}$$

where 2 is two quarks and m is gluons.

Next, we simplify the second stage by approximating $\frac{\bar{n}_g^h}{N_g} \approx \frac{\bar{n}_q^h}{N_q}$, assuming that the probabilities of hadron formation from a quark or gluon are equal. the parameter $\alpha = \frac{N_g}{N_q} = \frac{\bar{n}_g^h}{\bar{n}_q^h}$ to differentiate between hadron jets created from quarks or gluons in the second stage. We also simplify by setting N = N_q and $\bar{n}^h = \bar{n}_q^h$.

Thus,

$$Q_q^H = \left[1 + \frac{\bar{n}_p^h}{N_p}(z-1)\right]^N \tag{3.8}$$

$$Q_g^H = \left[1 + \frac{\bar{n}_p^h}{N_p}(z-1)\right]^{\alpha N} \tag{3.9}$$

By substituting (3.2) and (3.6) into (3.7) and taking the derivative with respect to z, we obtain the MD of hadrons produced in the e +e − annihilation process

$$P_n(s) = \Omega \sum_m P_m^P C_{(2+\alpha m)N}^n \left[\frac{\bar{n}^h}{\mathrm{N}}\right]^n \left[1 - \frac{\bar{n}^h}{\mathrm{N}}\right]^{(2+\alpha m)N-n} \tag{3.10}$$

where

$$P_m^P = \begin{cases} \left[\frac{k_p}{\bar{m}+k_p}\right]^{k_p}, & if\ m = 0 \\ \frac{k_p(k_p+1)\ldots(k_p+m-1)}{m!}\left[\frac{\bar{m}}{\bar{m}+k_p}\right]^m \left[\frac{k_p}{\bar{m}+k_p}\right]^{k_p}, & if\ m > 0 \end{cases} \tag{3.11}$$

The physical meaning of the six parameters, which we obtain because of fitting:

• Ω is normalization factor. In our case it should be equal to 2, since the experimental data are presented only for even n.

• $k_p = \frac{\tilde{A}}{A}$ is parameter showing how the quark bremsstrahlung process and the gluon fission process are related.

- $\overline{m}$ is the average multiplicity of gluons before hadronization.
- $\bar{n}_p^h$ is an average multiplicity formed from parton on the stage of hadronization.
- N_p is maximum secondary of hadrons from parton on the stage of hadronization.
- $\alpha = \frac{\bar{n}_g^h}{N_g} \approx \frac{\bar{n}_q^h}{N_q}$ is parameter entered so that there are fewer unknown parameters.

4. Sources

[1] QCD JETS AS MARKOV BRANCHING PROCESSES - A. Giovannini

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