Puzzles of Multiplicity (part 1)

Project of Interest, wave 11 (05.11-14.12.2024)

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Оглавление

1. Introduction

High-energy physics started with looking at multiparticle processes. As particle accelerators got more powerful, the understanding of many physics processes of nuclear physics and physics of elementary particles improved. This led to the development of different models to explain these processes.

One important thing scientists measure in experiments is called multiplicity, which is just the number of secondary particles produced. The multiplicity of charged particles is especially significant. For example, by studying events that produce a lot of secondary particles, we can better understand how hadrons interact, particularly during a phase called hadronization. The theory of high-energy physics, QCD, and another various models are being tested by examining multiparticle production (MPP). Since there are so many secondary particles, we use statistical methods to analyze MPP. The main statistical features we look at in multiplicity are its average value and variance.

To help with these studies, large particle accelerators like PEP, SLC, PETRA, TRISTAN, and LEP have been built to collide electrons and positrons at very high energies. The collisions between electron-positron pairs are interesting because their starting conditions are very clear. We know the total energy and momentum exactly, as well as other important details. This makes experiments with electron-positron annihilation great for studying strong interactions and testing QCD. Therefore, high-energy e+e annihilation is especially good for studying MPP, so that's what we will focus on in our work.

2. Theory

In this work we used idea of A. Giovannini for description of quark-gluon jets as Markov branching processes. He proposed to interpet the natural QCD evolution parameter Y for a theory with NC colours and Nf flavours, as the fickness of the jets and their development as Markov process. The Y parameter is as follows

$$Y = \frac{1}{2\pi b} log[1 + a_s b \cdot log(\frac{Q^2}{\mu^2})] \tag{2.1}$$

where $2\pi b = \frac{1}{6} \cdot (11_c - 2_f)$ for a theory with N_c colors and N_f flavors.

There are three main elementary processes that contribute to the overall gluon or quark distribution inside QCD jets with different weights:

1. $A\Delta Y-$ probability that one gluon in the infinitesimal interval Y will transform into two gluons
2. $\widetilde{A}\Delta Y-$ probability that a quark in the infinitesimal interval Y will radiate a gluon with the quark continuing on its original trajectory
3. $B\Delta Y-$ probability that a quark-antiquark pair in the infinitesimal interval Y will be created from a gluon

then we have such probabilities

1. A: gluon fission ($g \longrightarrow g + g$)
2. $\widetilde{A}$: quark bremsstrahlung ($q \longrightarrow q + g$)
3. B: quark pair creation ($q \longrightarrow q + \bar{q}$)

It is important to note that each individual parton acts independently of the others, always with the same infinitesimal probability. Also, $A, \bar{A}, B$ are assumed to be Y-independent constants.

The probability for a gluon or quark to convert into m_q quarks and m_g gluons in the interval $(Y, Y+\Delta Y)$ can be given by sum of probabilities:

$$\delta_{1,m_g}\delta_{0,m_q} + a^(g)_{m_g,m_q}\Delta Y + o(\Delta Y) forgluon \tag{2.2}$$

$$\delta_{0,m_g}\delta_{1,m_q} + a^(g)_{m_g,m_q}\Delta Y + o(\Delta Y) forquark \tag{2.3}$$

By the ways have a looks pay attentionthat $\delta_1, m_g)$ means that at the beginning of the process we have 1 gluon that converts into m_q.
Due to only 3 processes ($g \to g+g, q \to q+g, g \to q+\bar{q}$) are being allowed in the same interval ΔY, we get following possible transitions: a) for gluon:

$$1 + a^{(g)}_{1,0}\Delta Y + a^{(g)}_{2,0}\Delta Y + a^{(g)}_{0,2}\Delta Y + o(\Delta Y) \tag{2.4}$$

Where $a^{(g)}_{1,0} + a^{(g)}_{2,0} + a^{(g)}_{0,2} = 0, a^{(g)}_{1,0} < 0$ because of probability conservation, i.e., $a^{(g)}_{1,0} = -a^{(g)}_{2,0} = a^{(g)}_{0,2}$ b) for quark:

$$1 + a^{(q)}_{0,1}\Delta Y + a^{(q)}_{1,1}\Delta Y + o(\Delta Y) \tag{2.5}$$

Where $a^{(q)}_{0,1} + a^{(q)}_{1,1} = 0, a^{(q)}_{0,1} < 0$ because of probability conservation, i.e., $a^{(q)}_{0,1} = -a^{(q)}_{1,1}$ newline Let $a^{(g)}_{2,0} = A, a^{(g)}_{0,2} = B$ and $a^{(q)}_{1,1} = \widetilde{A}$. So we can write $1 - Au_g^2 - Bu_q^2$ and $1 - \widetilde{A}$ After this infinitesimal generating functions for quark and gluon jets are introduced:

$$\begin{aligned} w^{(g)}(u_g, u_q) = \sum_{m_g,m_q=0}^{\infty} a^{(g)}_{m_g,m_q} u_g^{m_g} u_g^{m_q} = (-A-B)u_g + Au_g^2 + Bu_q^2 w^{(q)}_{u_q,u_q} = \\ = \sum_{m_g,m_q=0}^{\infty} a^{(q)}_{m_g,m_q} u_g^{m_g} u_q^{m_q} = \widetilde{A}u_q + \widetilde{A}u_q u_g) \end{aligned} \tag{2.6}$$

Define $P_{m_g,m_q;n_g,n_q}(Y)$ as the probability that m_g gluons and m_q quarks will be transformed into n_g gluons and n_q quarks over a jeet of thickness Y. Probability generating function for a gluon jet and a quark over a jet of thickness Y. So the probability generating function for a gluon jet is

$$G(u_g, u_q; Y) = \sum_{n_g,n_q=0}^{\infty} P_{1,0;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} \tag{2.7}$$

and for a quark jet is

$$Q(u_g, u_q; Y) = \sum_{n_g,n_q=0}^{\infty} P_{0,1;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} \tag{2.8}$$

When considering the evolution of the total parton population (gluons and quarks) over a thickness Y, a probabilistic viewpoint enables us to consider this population as composed of independent sub-populations. Each of these sub-populations acts as if it originated from a single initial quark or gluon. Essentially, the overall evolution can be understood as the sum of independent parton populations, each beginning with one quark or gluon. This can be mathematically represented as follows:

$$\sum_{n_g,n_q}^{\infty} P_{m_g,m_q;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} = [G(u_g, u_q; Y)]^{m_g} [Q(u_g, u_q, Y)]_q^m \tag{2.9}$$

Since the process is homogeneous in Y the transition probabilities obey Chapman-Kolmogorov equations:

$$P_{m_g,m_q;n_g,n_q}(Y+Y) = \sum_{l_g,l_q=0}^{\infty} P_{m_g,m_q;l_g,l_q}(Y)P_{l_g,l_q;n_g,n_q}(Y) \tag{2.10}$$

Where the transition from one process to another is independent $l_g, l_q \to n_q n_q$ For a gluon jet, we get

$$P_{1,0;n_g,n_q}(Y+Y) = \sum_{l_g,l_q=0}^{\infty} P_{1,0;l_g,l_q}(Y)P_{l_g,l_q;n_g,n_q}(Y) \tag{2.11}$$

that means $1g \to n_g n_q$
and for a quark jet

$$P_{0,1;n_g,n_q}(Y+Y) = \sum_{l_g,l_q=0}^{\infty} P_{0,1;l_g,l_q}(Y)P_{l_g,l_q;n_g,n_q}(Y) \tag{2.12}$$

that means $1q \to n_g n_q$ So we can rewrite some equations:

$$\begin{aligned} G(u_g, u_q; Y+Y') &= \sum_{l_g,l_q=0}^{\infty} P_{1,0:l_g,l_q}(Y+Y)u_g^{l_g}u_q^{l_q} = \\ &= \sum_{l_g,l_q=0}^{\infty}\sum_{n_g,n_q=0}^{\infty} P_{1,0;n_g,n_q}(Y)P_{n_g,n_q;l_g,l_q}(Y)u_g^{l_g}u_q^{l_q} = \\ &= \sum_{n_g,n_q=0}^{\infty} P_{1,0;n_g,n_q}(Y)\sum_{l_g,l_q=0}^{\infty} P_{n_g,n_q;l_g,l_q}(Y)u_g^{l_g}u_q^{l_g} = \\ &= \sum_{n_g,n_q=0}^{\infty} P_{1,0;n_g,n_q}(Y)[G(u_g,u_q;Y)]^{n_g}[Q(u_g,u_q;Y)]^{n_q} \end{aligned} \tag{2.13}$$

$$G(u_g, u_q; Y+Y) = G[G(u_g,u_q;Y), Q(u_g,u_q;Y); Y] \tag{2.14}$$

We can write analogically probability generating function for quarks:

$$Q(u_g, u_q; Y+Y) = Q[G(u_g,u_q;Y), Q(u_g,u_q;Y); Y] \tag{2.15}$$

We can rewrite it using $P_{1,0;1,0}(\Delta Y) = 1, P_{0,1;0,1}(\Delta Y) = 1$:

$$G(u_g, u_q; Y+Y) = u_g + w^{(g)}(u_g,u_q)\Delta Y + o(Y) \tag{2.16}$$

$$Q(u_g, u_q; Y+Y) = u_q + w^{(q)}(u_g,u_q)\Delta Y + o(Y) \tag{2.17}$$

By substituting eq. 2.16 and eq. 2.17 into eq. 2.14 and eq. 2.15, replacing Y' with ΔY, then dividing both sides by ΔY and taking the limit as $\Delta Y \to 0$, we obtain

$$\frac{\partial G(u_g,u_q;Y)}{\partial Y} = \frac{\partial G}{\partial u_g}\cdot w^{(g)}(u_g,u_q) + \frac{\partial G}{\partial u_q}\cdot w^{(q)}(u_g,u_q) \tag{2.18}$$

$$\frac{\partial Q(u_g,u_q;Y)}{\partial Y} = \frac{\partial Q}{\partial u_g}\cdot w^{(g)}(u_g,u_q) + \frac{\partial Q}{\partial u_q}\cdot w^{(q)}(u_g,u_q) \tag{2.19}$$

After that we can get forward corresponding backward Kolmogorov equations follow from eq. 2.14 and eq. 2.15:

$$\frac{\partial G}{\partial Y} = w^{(g)}[G(u_g, u_q; Y), Q(u_g, u_q; Y)] \tag{2.20}$$

$$\frac{\partial Q}{\partial Y} = w^{(g)}[G(u_g, u_q; Y), Q(u_g, u_q; Y)] \tag{2.21}$$

Substituting eq. 2.6 into eq. 2.21 and eq. 2.22, we obtain

$$\frac{\partial G}{\partial Y} = -AG + AG^2 - BG + BQ^2 \tag{2.22}$$

$$\frac{\partial Q}{\partial Y} = -\widetilde{A}Q + \widetilde{A}GQ \tag{2.23}$$

3. Conclusion

Some Conclusions here.

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5. Bibliography

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