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Hard processes in high-energy factorization

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1 Abstract

In this paper, we review various approaches to the parton model in hard deep inelastic processes. The development of QCD to describe the above processes requires going beyond the standard CPM to take into account the transverse momentum and the virtuality of the initial partons, which is carried out within the framework of the kT-factorization or TMD-factorization models, but is rather limited. The need arises to construct a gauge-invariant calculation scheme in the kT-factorization model to describe multiscale rigid processes under Regge kinematics, which is implemented in the Reggeization of partons (RPA) approach based on the effective action of L.N. Lipatov.

2 Introduction

The ultrarelativistic proton is made up of a cloud of quarks, antiquarks and gluons, collectively called "partons". These partons arise due to the fact that the original quarks can emit gluons, which in turn can emit more gluons or split into quark-antiquark pairs. In parallel, the reverse process of fusion of partons also takes place, and as a result, a kind of equilibrium arises between partons of different types.

The processes of splitting and fusion of partons lead to the fact that their energies are not fixed, and with some probability can be any - from a certain minimum value and almost up to the energy of the entire proton. That is why one speaks not just of partons, but of parton densities: q (x), g (x), etc., where x is the fraction of the ultrarelativistic proton energy carried by a given parton.

Parton densities depend on one more variable - the scale of the process rigidity. The scale of stiffness, relatively speaking, shows at what distances the collision of partons occurs. The tougher the process, the more partons can take part in it, that is, the higher the parton density.

To study the parton structure of a proton, the processes of deep inelastic scattering are used. One of the main such processes is the process of inclusive deep inelastic scattering of a lepton by a proton

$$p(P) + e^{\pm}(q_2) \to e^{\pm}(q_3) + X,$$

where P, q_2, q_3 - are particle impulsess, and X - is an arbitrary hadronic final state.

Let's consider the kinematics of the parton model. The momentum of the parton under consideration is equal to $p = xP$, where P is the total momentum of the proton. We put k, k' - impulses of the incident and the scattered electron, respectively, then $q = k' - k$ is the transferred momentum, $p + q$ is the final momentum of the interacting quark

Then the Mandelstam variables $\widehat{s}$ and $\widehat{t}$ for the process of interaction of an electron with a quark inside a proton can be expressed as follows:

$$\widehat{s} = (p + k)^2 = 2(p \cdot k) = 2x(P \cdot k) = x(P + k)^2 = xS,$$
$$\widehat{t} = q^2$$

Then, for this process in the framework of the parton model, we obtain the scattering cross section

$$\frac{d^2\sigma}{dxdQ^2} = \sum_i f_i(x) Q_i^2 \frac{2\pi\alpha^2}{Q^4} \left[1 + \left(1 - \frac{Q^2}{xS} \right)^2 \right],$$

where Q_i is the charge of the $i-th$ quark, $Q^2 = -q^2$. And the structural functions of the proton take the form:

$$F_1(x) = \frac{1}{2x} F_2(x)$$
$$F_2(x) = \sum_i Q_i^2 x f_i(x)$$

3 Quantum chromodynamics

Since we are considering inelastic scattering by a proton, we will need some information about the interaction with the structural components of the proton (quarks). So let's start with an overview of the basics of interactions in QCD.

Quantum chromodynamics is used to describe the strong interaction of quarks and gluons. The fundamental parameters are the coupling constants $g_s \left(\alpha_s = \frac{g_s^2}{4\pi}\right)$ and quark masses. By analogy with quantum electrodynamics, the color charge is now used in quantum chromodynamics. QCD is a non-Abelian gauge theory based on the transformation group SU(3), whose dimension is 8, which corresponds to 8 gluons.

By requiring the observance of the local gauge symmetry of the free quark Lagrangian and adding the term of the free gluon field, we write the QCD Lagrangian

$$\mathcal{L} = \sum_q \bar{\psi}_{q,a} \left(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t_{ab}^C G_\mu^C - m_q \delta_{ab}\right) \psi_{q,b} - \frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu},$$

where γ^μ - Dirac matrix, $\psi_{q,b}$ - spinor of a quark field with flavor q and color a $\left(a = \overline{1, N_c}, N_c = 3\right)$, and mass m_q, G_μ^C operator which corresponds to the 4-vector of the gluon field, where C ranges from 1 to 8.

The symbol $F_{\mu\nu}^{A}$, represents the gauge invariant gluon field strength tensor, it is analogous to the electromagnetic field strength tensor, in quantum electrodynamics.

$$F_{\mu\nu}^{A} = \partial_{\mu} G_{\nu}^{A} - \partial_{\nu} G_{\mu}^{A} - g_s f_{ABC} G_{\mu}^{B} G_{\nu}^{C}, \quad \left[t^{A}, t^{B}\right] = i f_{ABC} t^{C},$$

where G_{μ}^{A}, are the gluon fields, dynamical functions of spacetime, in the adjoint representation of the SU(3) gauge group and f_{abc} are the structure constants of SU(3).

To describe the dependence of the effective color charge on the distance between the quarks, the parameter $\alpha_s(\mu_R^2)$ is introduced - the running coupling constant. As the distance between the quarks decreases, the coupling constant decreases (asymptotic freedom), and as the distance increases, it also increases (confinement).

Using the renormalization group equation and applying the conditions of our theory, as well as taking into account the relation for the mass scale in the massless quark approximation, we obtain the following expression for the equation of the coupling constant

$$\alpha_s\left(\mu_R^2\right) = \frac{12\pi}{(33 - n_f)\ln\left(\mu_R^2/\Lambda^2\right)}.$$

We can define the propagators by the relation where π_i presents any field. Curly, wavy and zigzag lines denote gluons, photons and weak bosons respectively, while full, dashed and dot lines stand for fermions (leptons and quarks), Higgs particles and ghosts fields, respectively. The vertices are derived using L_I , instead of usual usage of iL_I . All the momenta of the particles are supposed to flow in. The only exception was made for the ghost fields, where direction of momentum coincides with the direction of ghost number flow.

$$\Delta_{ij}(k) = i \int d^4x e^{-ikx} \langle 0 \left| T\left(q_i(x) q_j(0)\right) \right| 0 \rangle .$$

Due to the gauge invariance of the gluon field, we can determine it only up to the gauge transformation

$$G^a_\mu \to G'^a_\mu = G^a_\mu - \frac{1}{g_s} \partial_\mu \alpha_a - f_{abc} \alpha_b G^c_\mu$$

For further calculations, we need a propagator type that takes into account the rules for traversing the poles. Let us write the form of the propogator in the generalized Lorentz gauge

$$D^{\mu\nu} = \frac{1}{k^2 + i\varepsilon} \left[g^{\mu\nu} - (1 - \xi) \frac{k^\mu k^\nu}{k^2 + i\varepsilon} \right] .$$

Depending on the value of the parameter ε, we can get the form of our propagator in other gauges. $\varepsilon = 1$ in Feynman gauge and $\varepsilon = 0$ Landau gauge.

However, it should be understood that when choosing a generalized gauge, we may have divergences associated with virtual scalar and longitudinal gluons that do not contribute to physical states. To compensate for such divergences, the introduction of ghost particles is required.

But there is another way to solve this problem - this is the use of axial gauge (gauge vector n_μ is added), in which the Feynman rules do not contain Faddeev-Popov ghosts

$$D^{\mu\nu} = \frac{1}{k^2 + i\varepsilon} \left[g^{\mu\nu} - \frac{n_\mu k_\nu + k_\mu n_\nu}{(n \cdot k)} \right] .$$

In conclusion, we present the completeness relations for outer quarks and gluons, which will be necessary when summing over the polarizations in the squared moduli of the amplitudes. We choose the completeness relations for quarks in the form:

$$\sum_s u^s(p)\bar{u}^s(p) = \widehat{p} + m, \quad \sum_s v^s(p)\bar{v}^s(p) = \widehat{p} - m.$$

And also in the Feynman gauge, the completeness relation for gluon polarizations is:

$$\sum_\lambda \varepsilon^a_\mu(q)\varepsilon^{*b}_\nu(q) = -\delta^{ab} g_{\mu\nu}.$$

4 Collinear parton model

For the cross section for inclusive hard processes in hadronic collisions at high at energies, a factorization formula for the collinear parton model (CPM) is introduced

$$d\sigma = \sum_{ij} \int_0^1 dx_1 f_i\left(x_1, \mu^2\right) \int_0^1 dx_2 f_j\left(x_2, \mu^2\right) d\widehat{\sigma}^{ij}_{CPM}\left(x_1, x_2, \mu^2\right).$$

The slow evolution of parton densities is determined by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations, which describe the evolution of the densities $f_g(x, \mu^2)$ and $f_q(x, \mu^2)$ for quarks and antiquarks of each aroma. These equations are as follows:

$$\frac{d}{d\ln\mu^2} f_g\left(x, \mu^2\right) = \frac{\alpha_s\left(\mu^2\right)}{2\pi} \int_x^1 \frac{dz}{z} \left\{ P_{g\leftarrow q}(z) \sum_q \left[f_q\left(\frac{x}{z}, \mu^2\right) + f_{\bar{q}}\left(\frac{x}{z}, \mu^2\right)\right] + \right.$$
$$\left. + P_{g\leftarrow g}(z) f_g\left(\frac{x}{z}, \mu^2\right) \right\}$$

$$\begin{array}{c} \frac{d}{d\ln\mu^2} f_q\left(x,\mu^2\right) = \frac{\alpha_s\left(\mu^2\right)}{2\pi}\int_x^1 \frac{dz}{z}\left\{P_{q\leftarrow q}(z) f_q\left(\frac{x}{z},\mu^2\right) + P_{q\leftarrow g}(z) f_g\left(\frac{x}{z},\mu^2\right)\right\}, \\ \frac{d}{d\ln\mu^2} f_{\bar{q}}\left(x,\mu^2\right) = \frac{\alpha_s\left(\mu^2\right)}{2\pi}\int_x^1 \frac{dz}{z}\left\{P_{q\leftarrow q}(z) f_q\left(\frac{x}{z},\mu^2\right) + P_{q\leftarrow g}(z) f_g\left(\frac{x}{z},\mu^2\right)\right\} \end{array}.$$

The splitting functions P have the following physical meaning: $P_{q\leftarrow q}(z)$ is the probability that the gluon was emitted by a quark and carries its fraction of the momentum z; $P_{g\leftarrow q}(z)$ is the probability of a gluon emitting a gluon with a fraction of the momentum z; $P_{q\leftarrow g}(z)$ is the probability that a quark will emit a gluon and therefore will have a fraction of the momentum z; $P_{g\leftarrow g}(z)$ - the probability of the production of a pair by a gluon while the quark carries away the z fraction of the gluon momentum. These functions look like this:

$$\begin{array}{c} P_{q\leftarrow q}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right] \\ P_{g\leftarrow q}(z) = \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right] \\ P_{q\leftarrow g}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right] \end{array}$$

$$P_{g\leftarrow g}(z) = 6\left[\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) + \left(\frac{11}{12} - \frac{n_f}{18}\right)\delta(1-z)\right],$$

where nf is the number of flavors of light quarks, and the function $z\frac{1}{(1-z)_+}$ - the so-called "plus-replacement defined so that the relation

$$\int_0^1 dz\frac{f(z)}{(1-z)_+} = \int_0^1 dz\frac{f(z)-f(1)}{(1-z)}.$$

5 Reggeization approach of partons

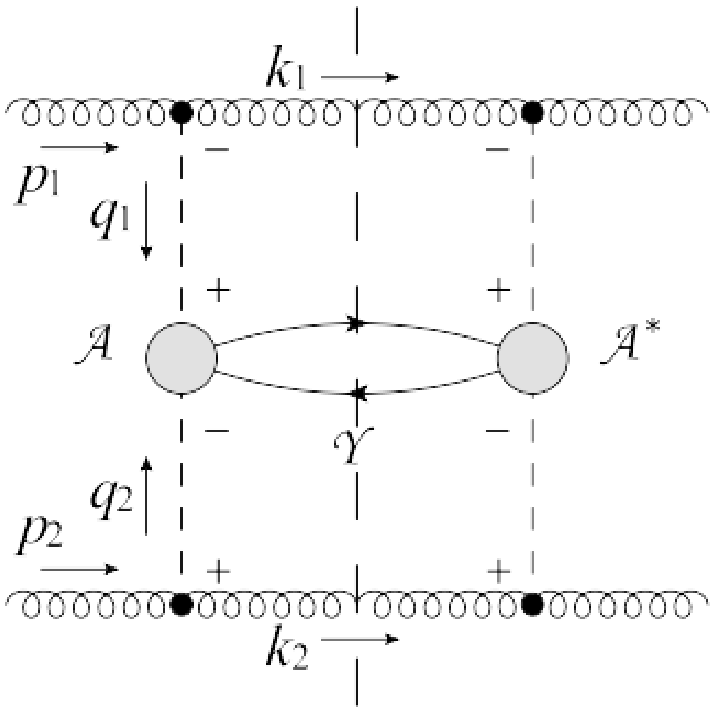
The reggeization of partons approach is a gauge-invariant scheme of quantum chromodynamics at high energies, which extends the collinear parton model to the case of hard multiscale processes in the high-energy regime when we are dealing with processes in multi-Regge kinematics. In the MRC, the collision energy $\sqrt{S}$ is quite large compared to the transverse momenta of the final particles $\sqrt{S} \gg |kT_i|$ and the invariant masses of their pairs $M_{ij} = p(k_i + k_j)^2$. The main component of the PRA at high energies is the kT -factorization of non-integrated parton distribution functions (nPDF) and gauge-invariant parton cross section with virtual partons in the initial state, where these partons are considered as Reggeized gluons R and Reggeized quarks Q.

$$g\,(p_1) + g\,(p_2) \to g\,(k_1) + \mathcal{Y}\,(P_{\mathcal{A}}) + g\,(k_2)\,.$$

In the collinear limit, where $k_{T1,2}^2 \ll \mu^2$ and $0 \leq z_{1,2} \leq 1$, the asymptotics for the square of the tree matrix element of the process is well known

$$\overline{|\mathcal{M}|^2}^2_{\text{C.L.}} \simeq \frac{4g_s^4}{\mathbf{k}_{T1}^2\mathbf{k}_{T2}^2} P_{gg}\,(z_1)\, P_{gg}\,(z_2)\, \frac{\overline{|\mathcal{A}_{CPM}|^2}}{z_1 z_2}.$$

The MRC asymptotics of the amplitudes in the EFT are constructed from gauge-invariant blocks - effective vertices that describe the production of clusters of QCD partons, strongly separated from each other in rapidity. These effective vertices are linked together by t-channel exchanges of gauge-invariant virtual degrees of freedom - reggeized glues $R_\pm$ and reggeized quarks $Q_\pm$. The latter obey special kinematic constraints, since the fields $Q_\pm(R_\pm)$ carry only the $q_\pm$ conical component of the momentum



and the transverse momentum of the same order, while q = 0. These kinematic constraints are equivalent to the MRC. The requirements for the gauge invariance of the effective vertices and the aforementioned kinematic constraints on the interaction of QCD partons and Reggeons in the EFT are not local and contain the Wilsonian exponents of gluon fields. After expansion in terms of perturbation theory, the latter generate infinite series of induced vertices of interactions between partons and reggeons

Effective vertex $R_{\pm}gg$, shown as a diagram in the figure

$$\Gamma^{abc}_{\mu\nu\pm}(k_1, k_2) =$$

$$= -ig_s f^{abc}\left[2g_{\mu\nu}k_1^{\mp} + (2k_2+k_1)_\mu n_\nu^{\mp} - (2k_1+k_2)_\nu n_\mu^{\mp} - \frac{(k_1+k_2)^2}{k_1^{\mp}} n_\mu^{\mp} n_\nu^{\mp}\right].$$

Calculating the square of the effective vertex $R_{\pm}gg$, folded with the polarization vectors of real external gluons, we get:

$$\sum_{\lambda_1,\lambda_2} \left|\Gamma_{\mu\nu\pm}(k_1,-k_2)\,\epsilon_\mu(k_1,\lambda_1)\,\epsilon_\nu^\star(k_2,\lambda_2)\right|^2 = 8\left(k_1^{\mp}\right)^2.$$

For the square of the amplitude of the process, written using the Feynman rules, the following condition for the collinear limit for the RAP amplitude must be satisfied

$$\int \frac{d\phi_1 d\phi_2}{(2\pi)^2} \lim_{t_{1,2}\to 0} \overline{\mathcal{A}_{PRA}|}^2 = \overline{|\mathcal{A}_{CPM}|}^2.$$

Using the result and Feynman's rules (for this case), we can obtain the MRK asymptotics of the square of the process amplitude. And now we introduce the modified MRK approximation (mRMK) for the squared modulus of the amplitude of the subprocess

$$\overline{|\mathcal{M}|^2}_{\text{mMRK}} \simeq \frac{4g_s^4}{q_1^2 q_2^2} P_{gg}(z_1)\, P_{gg}(z_2)\, \frac{\overline{|\mathcal{A}_{PRA}|^2}}{z_1 z_2}.$$

To derive the PDP factorization formula in LO, we substitute the mMCR approximation into the CPM factorization formula, integrating additional partons $k_{1,2}$ over the phase space:

$$d\sigma = \int \frac{dk_1^+ d^2\mathbf{k}_{T1}}{(2\pi)^3 k_1^+} \int \frac{dk_2^- d^2\mathbf{k}_{T2}}{(2\pi)^3 k_2^-} \int d\tilde{x}_1 d\tilde{x}_2 f_g\left(\tilde{x}_1, \mu^2\right) f_g\left(\tilde{x}_2, \mu^2\right) \frac{\overline{|\mathcal{M}|^2}}{2S\tilde{x}_1\tilde{x}_2} \times$$
$$\times (2\pi)^4 \delta\left(\frac{1}{2}\left(q_1^+ n_- + q_2^- n_+\right) + q_{T1} + q_{T2} - P_{\mathcal{A}}\right) d\Phi_{\mathcal{A}} .$$

The final form of our non-integrated parton distribution functions

$$\Phi_i\left(x, t, \mu^2\right) = T_i\left(t, \mu^2\right) \frac{\alpha_s(t)}{2\pi} \sum_{j=q,\bar{q},g} \int_x^1 dz P_{ij}(z) \frac{x}{z} f_j\left(\frac{x}{z}, \mu^2\right) \theta\left(1 - \Delta_{KMR}\left(t, \mu^2\right) - z\right)$$

The collinear singularity is regularized by the Sudakovskiy form factor, which resumulates the two-logarithmic corrections in the leading logarithmic approximation in a manner similar to that used in standard parton shower algorithms

$$T_i\left(t, \mu^2\right) = \exp\left[-\int_t^{\mu^2} \frac{dt'}{t'} \frac{\alpha_s\left(t'\right)}{2\pi} \sum_{j=q,\bar{q},g} \int_0^1 dz z \cdot P_{ji}(z) \theta\left(1 - \Delta_{KMR}\left(t', \mu^2\right) - z\right)\right].$$

Non-integrated parton distribution functions satisfies the following normalization condition

$$\int_0^{\mu^2} dt \Phi_i\left(x, t, \mu^2\right) = x f_i\left(x, \mu^2\right),$$

which provides normalization for one-scale observables in a rigid process to the corresponding LO CPM results up to power-law suppressed corrections and NLO terms of degree α_s. The results for multiscale observables in the

RPA differ significantly from those obtained in the CPM due to nonzero transverse momenta of partons in the initial state.

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